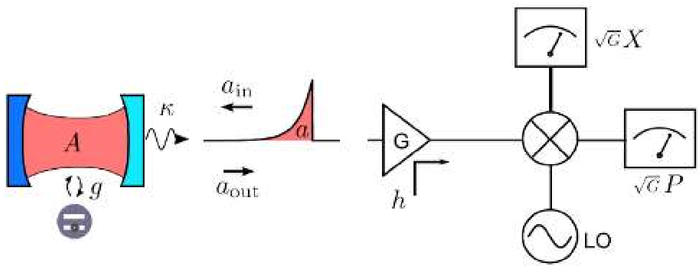
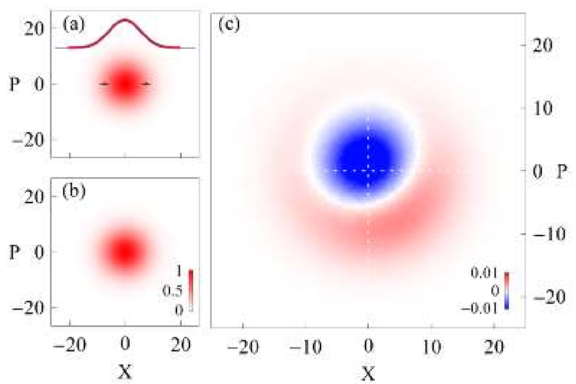
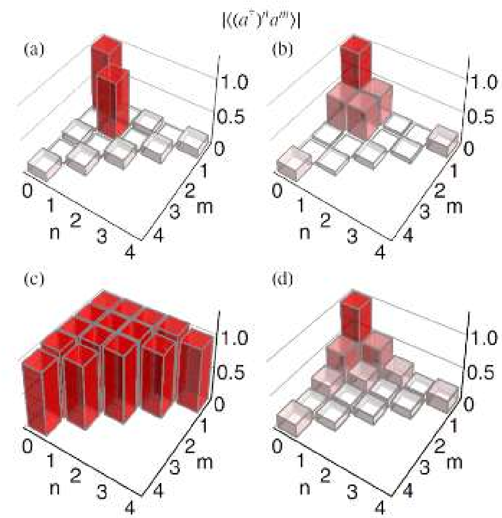
# Single photon sources

Monday, March 7, 2016 8:40 AM




Eichler, C., et al PRL 106.22 (2011): 220503.




Photon states created inside a cavity do not stay there forever: they leak out.

If we conduct those states through 'open' microwave lines we may begin to study a kind of "quantum microwave photonics" that excels in

(1) State preparation, (in cavities) (2) strong light-matter interactions (and long coherence times) and (3) good measurement schemes (i.e. not single shot)

# Open transmission line

Saturday, March 5, 2016 1:36 PM

$$H = \sum_k \hbar\omega_k \left(a_k^\dagger a_k + \tfrac{1}{2}\right) \quad \text{with fields} \quad \begin{cases} \phi(x) = \sum_n \dfrac{\phi_n(x)}{\sqrt{\ell}} \cdot \sqrt{\dfrac{\hbar Z}{|k_n|}}\, \dfrac{(a + a^\dagger)}{\sqrt{2}} \\[2ex] q(x) = \ell \underbrace{\sum_n \dfrac{\phi_n(x)}{\sqrt{\ell}} \sqrt{\dfrac{\hbar |k_n|}{Z}}\, \dfrac{i}{\sqrt{2}} (a^\dagger - a)}_{p(x)} \end{cases}$$

Where we assumed:

- uniform transmission line
- $c_0, l_0$ : capacitance/inductance per unit length
- Any boundary conditions, length $\ell$, $\phi(x) \in \mathbb{R} \sim \cos/\sin\,(k_n x)$
- Also uniform impedance $Z = \sqrt{l_0/c_0}$

Open boundary conditions

$$\phi(x) = \sum_n \frac{1}{\sqrt{\ell}} \sqrt{\frac{\hbar Z}{|k_n|}} \left(a e^{ik_n x} + a^\dagger e^{-ik_n x}\right)$$

# Qubit on the line

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a) Capacitive coupling:

$$H_{int} = \frac{1}{C_\Sigma} \underset{\text{qubit charge}}{Q} \cdot \overbrace{C_g \underset{\text{voltage on line}}{\partial_t \phi(x)}}^{\text{induced charge}} \sim \frac{C_g}{C_\Sigma} Q \cdot \left(-i \left[\phi(x), H\right]\right) = \frac{C_g}{C_\Sigma} Q \sum_n \sqrt{\frac{\hbar Z}{2 l k_n}} \left(-i\omega_n a_n e^{ik_n x} + i\omega_n e^{-ik_n x} a_n^\dagger\right)$$

$$= \sigma^x \sum_n \left(g_n^{cap} a_n^\dagger + \bar{g}_n^{cap} a_n\right)$$

$$g_n^{cap} = \frac{e^{-ik_n x}}{\sqrt{l}} \, i \sqrt{\frac{\hbar\omega_n}{2C_0}} \, \frac{C_g}{C_\Sigma}$$

b) Inductive coupling

$$I_{cav} \sim \frac{1}{L_0} \frac{\left(\phi(x_{i+1}) - \phi(x_i)\right)}{\Delta x} \sim \frac{1}{L_0} \partial_x \phi \sim \frac{1}{L_0} \cdot (ik_n) \phi_n(x)$$

$$H_{int} \sim M I_{qb} I_{cav} \sim M I_{qb} \frac{1}{L_0} \sum_n \sqrt{\frac{\hbar Z}{2 l k_n}} \left(ik_n a_n e^{ik_n x} - ik_n a_n^\dagger e^{-ik_n x}\right)$$

$$g_n^{ind} \sim M I_c \frac{e^{-ik_n x}}{\sqrt{l}} \cdot \sqrt{\frac{\hbar\omega_n}{L_0}}$$

# Ohmic spin-boson model



Both for capacitive and inductive models we can write

$$H = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon}{2}\sigma^x + \sum_n \sigma^x\left(g_n a_n^+ + \bar{g}_n a_n\right) + \sum_n \omega_n a_n^+ a_n$$

where in both cases

$$\omega_n \sim v_g k_n \quad \text{and} \quad g_n \sim \sqrt{\frac{\omega_n}{l}} \times g$$

We introduce the spectral function

$$J(\omega) = 2\pi \sum_n |g_n|^2 \delta(\omega-\omega_n) = |g|^2 \sum_k \frac{2\pi}{l} \cdot \omega_K \, \delta(\omega-\omega_n)$$

$$\simeq \int_{\mathbb{R}} |g|^2 \omega_n \, \delta(\omega-\omega_n)\, dk = \frac{2|g|^2}{v_g} \cdot \int \omega_n \, \delta(\omega-\omega_n)\, d\omega_n = \frac{2|g|^2}{v_g}\omega \equiv \pi\alpha\omega$$

# Spontaneous emission

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We will work in a regime of not too fast qubit dynamics, with no bias field $\varepsilon$, and apply the RWA

$$H \sim \Delta |e\rangle\langle e| + \sum_k \left( g_k \sigma^- a_k^+ + h.c. \right) + \sum_k \omega_k a_k^+ a_k$$

Our trial wavefunction

$$|\psi(t)\rangle = e(t)\,|e,0\rangle + \sum_n \phi_n(t)\,|g, k_n\rangle$$

$$i\partial_t e = \Delta e + \sum_n g_n^* \phi_n$$

$$i\partial_t \phi_n = \omega_n \phi + g_n e$$

We integrate the second one

$$\phi_n(t) = e^{-i\omega_n t}\,\phi_n(0) - i \int_0^t e^{-i\omega_n (t-\tau)} g_n e(\tau)\, d\tau$$

# Memory Function

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We can now feed this into the qubit using $e(t) \sim e^{-i\Delta t}\,\xi$

$$\partial_t \xi = -i \sum_n g_n^* e^{-i[\omega_n - \Delta]t} \phi(\omega) + \int_0^t k(t-\tau)\,\xi(\tau)\,d\tau$$

$$k(t-\tau) = \sum_n |g_n|^2 e^{-i[\omega_n - \Delta](t-\tau)} = \text{memory function}$$

Note how $J(\omega)$ appears

$$k(t-\tau) = \frac{1}{2\pi}\int_{\mathbb{R}} d\omega \left[2\pi \sum_n |g_n|^2 \delta(\omega - \omega_n)\right] e^{-i(\omega - \Delta)(t-\tau)} = \frac{1}{2\pi}\int_{\mathbb{R}} d\omega\, J(\omega)\, e^{-\cdots}$$

We expect a) $k(t)$ is nonzero over a time $T_{mem} \ll$ the time for $\xi(\tau)$ to change

b) $\int_0^t k(t-\tau)\,\xi(\tau)\,d\tau \sim \xi(t)\int_0^t k(t-\tau)\,d\tau \sim \xi(t)\int_{+\infty}^0 k(\tau)\,d\tau$

# Example: exponential cut-off

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We assume $J(\omega) \sim \pi\alpha\omega \cdot e^{-\omega/\omega_c}$, which cancels the action of highly energetic photons

a) Their impedance match to the system is bad

b) $g_k$ actually has some width, related to the size of qubit

In this case

$$k(t) = \frac{1}{2\pi}\int_0^\infty d\omega\, \pi\alpha\omega\, e^{-\omega/\omega_c - i[\omega-\Delta]t} = \frac{-1}{T_m}\frac{\partial}{\partial T_m} W(t) = \frac{\alpha}{2T_m^3}\frac{e^{i\Delta t}}{(1+it/T_m)^2}$$

$$W(t) = \int_0^\infty d\omega\, \frac{\alpha}{2}\, e^{-\omega T_m - i[\omega-\Delta]t} = \frac{\alpha e^{i\Delta t}/2}{-it - T_m} \qquad \frac{-1}{T_m}\frac{\partial}{\partial T_m} W = \frac{-1}{T_m}\frac{-\alpha e^{+i\Delta t}/2}{(it+T_m)^2}$$

so indeed the system has a very short memory time $T_m \sim 1/\omega_c \ll 1/\Delta$

# Markov approximation

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$$\partial_t \xi = \xi_{ext}(t) - \xi(t) \int_0^\infty k(\tau)\, d\tau$$

← Markov approximation: system does not change significantly within memory time

$$\int_0^\infty k(\tau)\, d\tau = \int_0^\infty \frac{1}{2\pi} \int_{\mathbb{R}} J(\omega)\, e^{-i[\omega-\Delta]\tau}\, d\omega\, d\tau = \frac{1}{2} J(\Delta) - i\, \frac{PV}{2\pi} \int \frac{J(\omega)}{\Delta-\omega}\, d\omega$$

$$\int_0^\infty e^{\pm i\omega\tau}\, d\tau = \pi\,\delta(\omega) \pm i\, PV\left(\frac{1}{\omega}\right)$$

$$\partial_t \xi = \left[-\frac{\gamma}{2} - i\delta\right] \xi + \phi_{ext}$$ where $\gamma = J(\Delta)$ and the Lamb shift

$$\delta = \frac{1}{2\pi} \lim_{\varepsilon\to 0} \left\{ \left[ \int_{-\infty}^{\Delta-\varepsilon} + \int_{\Delta+\varepsilon}^{+\infty} \right] \frac{J(\omega)}{\omega-\Delta}\, d\omega \right\} \sim 0 \text{ if } J(\omega) \sim \text{constant}$$

# Source terms

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$$\xi_{ext}(t) = -i \sum_k g_k^* e^{-i[\omega_k - \Delta]t} \phi_k(\omega)$$

Using $J(\Delta) \sim \gamma$ we can revisit the value of "$g_k$" and do a Markov approximation

$$J(\omega) = \sum_k 2\pi |g_k|^2 \delta(\omega - \omega_k) \approx \sum_k \underbrace{\Delta k}_{\frac{2\pi}{L}} . |g|^2 \omega_k \delta(\omega - \omega_k) = \frac{2g^2}{v_g} . \omega$$

$$g_k \sim \frac{e^{ikx_{gb}}}{\sqrt{L}} . g\sqrt{\omega_k} \qquad \sum_k \Delta k = 2\int \frac{d\omega}{v_g}.$$

$$J(\Delta) = \gamma \Rightarrow g = \sqrt{\frac{v_g \gamma}{2\Delta}}$$

$$\xi_{ext}(t) = -i \sum_k \sqrt{\frac{v_g \gamma}{2}} . \underbrace{\cancel{\sqrt{\frac{\omega_k}{\Delta}}}}_{1} \frac{e^{ikx - i\omega_k t}}{\sqrt{L}} \phi_k(\omega) e^{+i\Delta t} \sim -i e^{i\Delta t} \sqrt{\frac{v_g \gamma}{2}} \Phi(x,t)$$

where $\Phi(x,t)$ is simply $\langle x | \Psi \rangle$ the wavefunction of the photon at that point

# Input fields

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We split the waves into right- and left-moving fields

$$\phi_\pm(x,t) = \sum_{k\in\mathbb{R}^+} \frac{e^{\pm ikx}}{\sqrt{l}} \phi_k(t)$$

$$= \sum_{k\in\mathbb{R}^+} \frac{e^{\pm ikx - i\omega_k(t-T)}}{\sqrt{l}} \psi_k(T) - i \int_T^t d\tau \sum_{k\in\mathbb{R}^+} \frac{e^{\pm ikx}}{\sqrt{l}} g_{\pm k} e(\tau)\, e^{-i\omega_k(t-\tau)}$$

Using $g_k = \frac{e^{-ikx_{gb}}}{\sqrt{l}} \sqrt{\frac{v_g \gamma}{2}}$

$$\varphi_\pm(x,t) = \underbrace{\phi_\pm^{in}(x,t)}_{\text{definition}} - i \int_{T_{-\infty}}^t d\tau \underbrace{\sum_{k\in\mathbb{R}} \frac{1}{l}}_{\sim \frac{1}{2\pi}\int_0^\infty dk \,\sim\, \frac{1}{2\pi v_g}\int_0^\infty d\omega_k} e^{\pm ik(x-x_{gb})} \sqrt{\frac{\gamma}{2v_g}}\, e(\tau)\, e^{-i\omega_k(t-\tau)}$$

$$\int_0^\infty e^{i\omega_k(\pm x/v_g + \tau)} \sim \pi\,\delta(\dots)$$

$$= \phi_\pm^{in}(x,t) - \underbrace{i\,\frac{1}{2}\sqrt{\frac{\gamma}{ev_g}}\, e(t \mp (x-x_{gb})/v_g)}_{\text{only when } \pm\delta x/v_g \,\in\, [T_{-\infty}, t]} \exp\left(\pm ik(x-x_{gb})\right)$$

# Input-output relations

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$$\phi_\pm(x,t) = \phi^{in}_\pm(x, t \pm x/v_g) - i\,\frac{1}{2}\sqrt{\frac{v_g\gamma}{2}}\, e(t)\, \exp(\pm i k x) \qquad x_{qb}=0$$

$$= \phi^{out}_\pm(x, t \pm x/v_g) + i\,\frac{1}{2}\sqrt{\frac{v_g\gamma}{2}}\, e(t)\, \exp(\pm i k x)$$

$$\phi^{out} \sim \phi^{in} - i\sqrt{\frac{v_g\gamma}{2}}\, e(t)$$ input-output relations b/w. incoming and scattered fields

This can be used to recast the qubit equation

$$\partial_t e(t) = (-i\Delta - \gamma/2)\, e(t) - i\sqrt{\frac{v_g\gamma}{2}}\left[\phi^{in}_+(0,t) + \phi^{in}_-(0,t)\right]$$

where we have undone the change of variables $e(t) = e^{-i\Delta t}\xi(t)$

# Continuous wave drive

Saturday, March 5, 2016 4:18 PM

We can integrate the qubit dynamics

$$e(t) = e^{[-i\Delta - \gamma/2]t} e(0) - i\sqrt{\frac{v_g \gamma}{2}} \int_{T_{-\infty}}^{t} d\tau \, e^{[-i\Delta - \gamma/2](t-\tau)} \left[ \phi_+^{in}(0,t) + \phi_-^{in}(0,t) \right]$$

We assume some slowly growing input states in the past

$$\phi_\pm^{in}(x_{qb}, t) = \Theta(-t) \, a_\pm \, e^{(-i\omega_h + 0^+)t} = \begin{cases} a_\pm \exp(-i\omega_h t + 0^+ t) & t < 0 \\ 0 & \text{else} \end{cases}$$

$$\int_{T_{-\infty}}^{t} d\tau \, e^{[i\Delta - \gamma/2](t-\tau)} e^{(-i\omega_h + 0^+)\tau} [a_+ + a_-] \, \Theta(-\tau) \overset{t<0}{=} \frac{e^{(-i\omega_h + 0^+)t} (a_+ + a_-)}{i(\Delta - \omega_h) + \gamma/2 + \cancel{0^+}}$$

$$\overset{t \geq 0}{=} \frac{e^{(-i\Delta - \gamma/2)t}}{i(\Delta - \omega_h) + \gamma/2 + 0^+} (a_+ + a_-)$$

# Scattering relations

Sunday, March 6, 2016 9:31 AM

$$\phi_\pm^{out}(x,t) = \phi_\pm^{in}(x,t) - i\sqrt{\frac{\gamma}{2v_g}}\, e(t \mp x/v_g)\, e^{\pm ikx}$$

note that the qubit emits in both directions even if one has no photons, say $a_- = 0$

We can expand $\phi_\pm^{out}(x,t) \sim b_\pm \exp(\pm ikx - i\omega_n t)$

$$b_+ = a_+ - i\sqrt{\frac{\gamma}{2v_g}} \cdot \left[ -i\sqrt{\frac{v_g\gamma}{2}}\, \frac{a_+ + a_-}{i(\Delta-\omega_n) + \gamma/2} \right]$$

$$\begin{pmatrix} b_+ \\ b_- \end{pmatrix} = \begin{pmatrix} 1+R_\omega & R_\omega \\ R_\omega & 1+R_\omega \end{pmatrix} \begin{pmatrix} a_+ \\ a_- \end{pmatrix}$$

$R_\omega = -\frac{\gamma}{2}\frac{1}{i[\Delta-\omega]+\gamma/2}$ reflection &

$T_\omega = 1 + R_\omega$ transmission amplitude

# Extensions

Sunday, March 6, 2016 9:50 AM

Dephasing and dissipation are taken into account in an "additive" way, complicating the equation for the qubit, and converting it into a master equation.

The outcome reads

$$R = -\left[\eta \frac{\Gamma_1}{2\Gamma_2}\right] \frac{1 + i\,\frac{\Delta - \omega_2}{\Gamma_2}}{1 + \left(\frac{\Delta - \omega_2}{\Gamma_2}\right)^2 + \frac{\Omega^2}{\Gamma_1 \Gamma_2}} \quad , \quad T = 1 + R$$

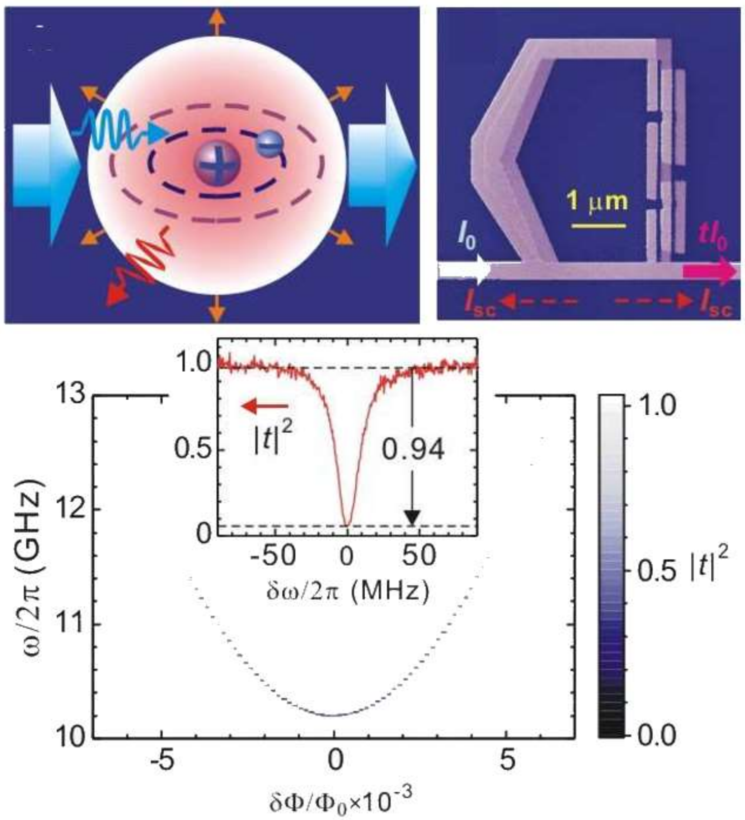
where $\Gamma_1 = \gamma$ = relaxation rate

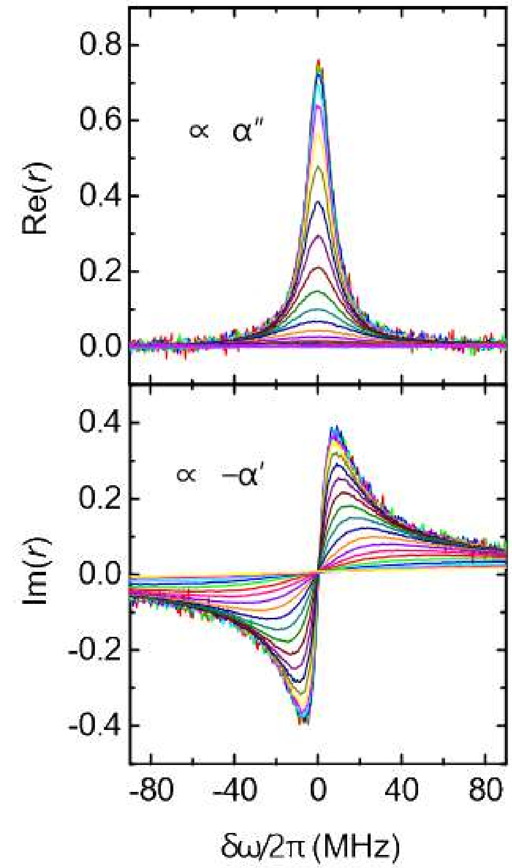
$\Gamma_2 = \frac{\gamma}{2} + \gamma_\phi$ = total dephasing rate

$\eta \Gamma_1$ = fraction of released energy that goes into the waveguide $\approx \Gamma_1$

# Experiment with a qubit

Sunday, March 6, 2016 9:58 AM

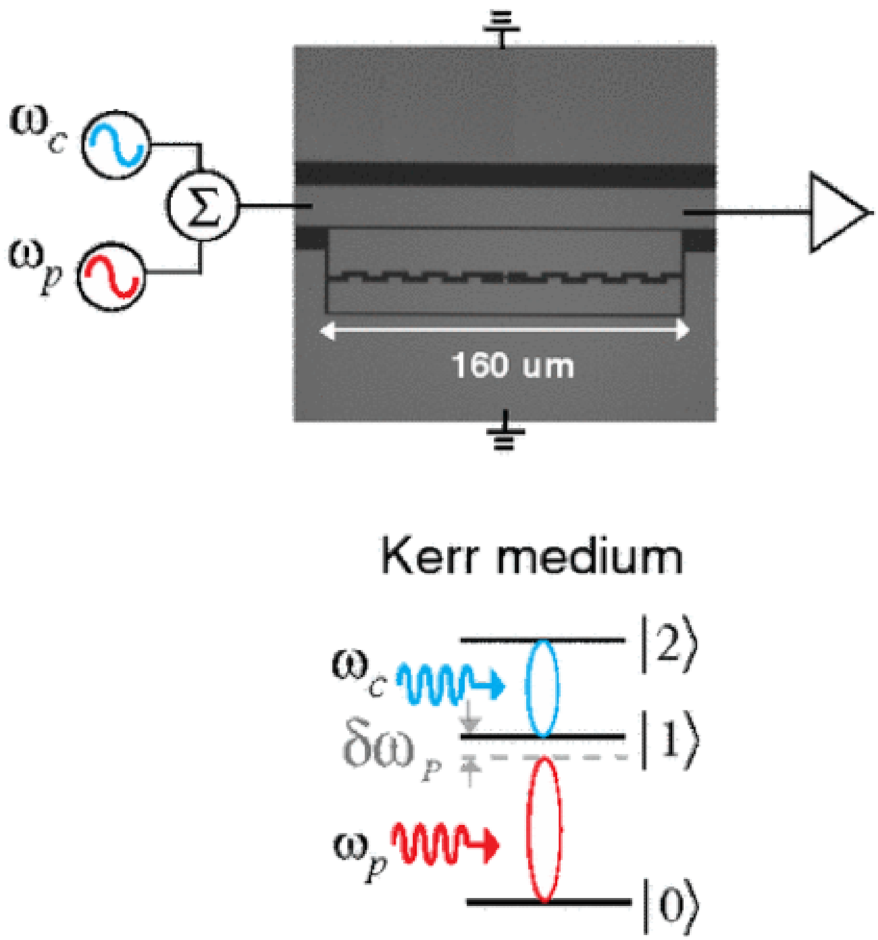


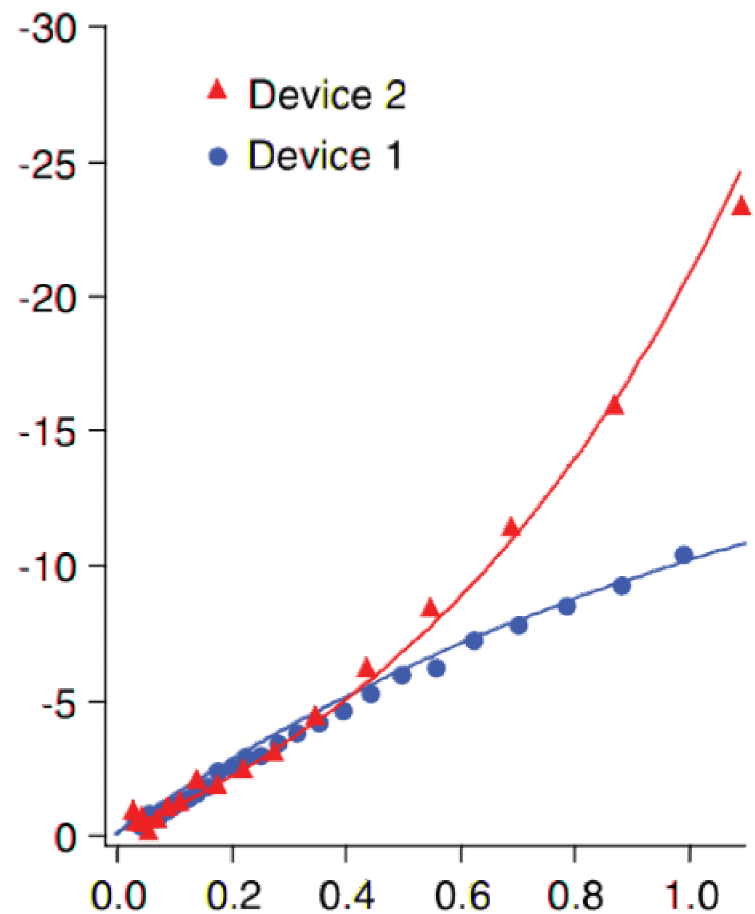




Astafiev, O., et al. "Resonance fluorescence of a single artificial atom." Science 327.5967 (2010): 840-843.

# Cross-kerr effect

Sunday, March 6, 2016 10:04 AM



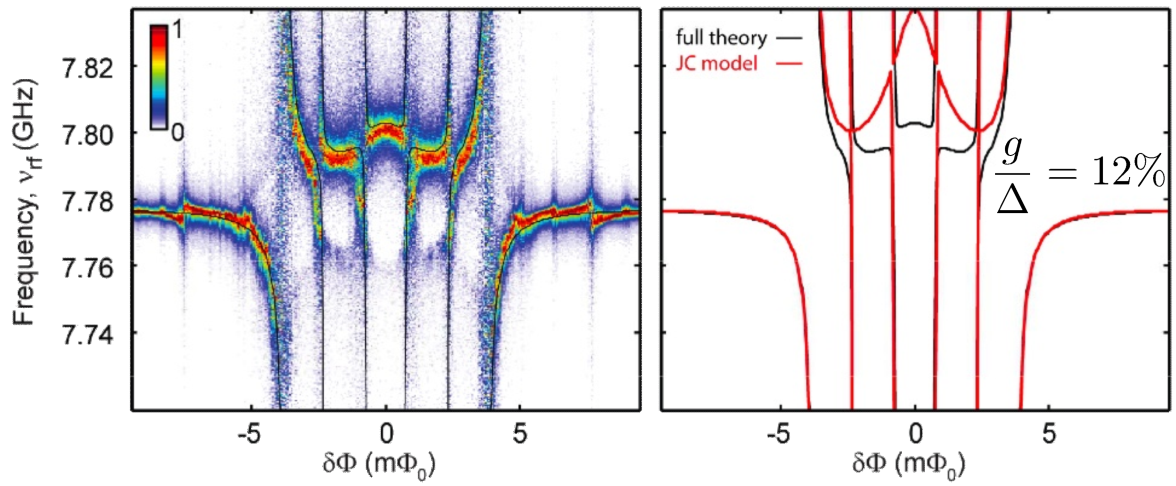




Io-Chun Hoi et al
Phys. Rev. Lett. 111, 053601 (2013)

# Ultrastrong coupling

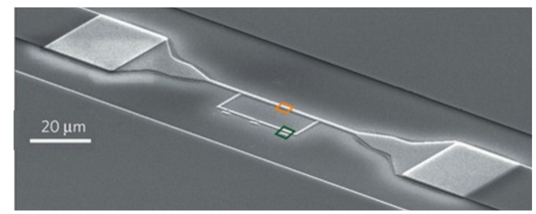
Sunday, March 6, 2016 11:00 AM

When $g \sim \Delta, \omega$, we cannot apply the RWA because Rabi oscillations become comparable to free dynamics, and $\Delta \pm \omega$ terms are no longer negligible




**RWA** $\quad g(\sigma^+ a + \sigma^- a^\dagger)$

**Rabi** $\quad g\sigma^x(a + a^\dagger)$




*T. Niemczyk et al, Nature Physics* **6***,772–776 (2010)*

This leads to interesting physics, which ranges from dressed states for the qubits to ultrafast quantum operation. What would be the equivalent in the continuum? We only have two timescales there

$$\gamma \sim \Delta$$

# Relation between cavities and waveguides

Sunday, March 6, 2016 10:23 AM

Starting point
$$\begin{cases} H = \dfrac{\Delta}{2}\sigma^z + \dfrac{\varepsilon}{2}\sigma^x + \sum_n \sigma^x (g_n a_n^+ + \bar{g}_n a_n) + \sum_n \omega_n a_n^+ a_n \\ \omega_n \sim v_g k_n \quad \text{and} \quad g_n \sim \sqrt{\omega_n}\, \chi \dfrac{\phi(x)}{\sqrt{\ell}} \end{cases}$$

$\chi$ → proportionality constant w. all "microscopic"

$\dfrac{\phi(x)}{\sqrt{\ell}}$ → normalized mode

a) Resonant cavity : $\ell = \lambda/2$

$$\bar{g}_0 = \sqrt{\Delta}\ \chi \cdot \frac{2}{\sqrt{\lambda}}$$ ← normalization of $\cos(k_\Delta x)$

$$\Delta = v_g \cdot k_\Delta = v_g \frac{2\pi}{\lambda} \Rightarrow \frac{1}{\lambda} = \frac{\Delta}{2\pi v_g} \Rightarrow \bar{g}_0 = \sqrt{\frac{\Delta^2}{2\pi v_g}} \cdot \chi \cdot 2 = \Delta \sqrt{\frac{2}{\pi v_g}}\ \chi$$

or alternatively $\chi = \left(\dfrac{\bar{g}_0}{\Delta}\right) \cdot \sqrt{\dfrac{v_g \pi}{2}}$ is proportional to the ratio between Rabi frequency and qubit/cavity frequency in a c.QED experiment

# Continuum limit

Sunday, March 6, 2016 10:49 AM

b) Continuum: $g_k = \sqrt{\omega_k} \times \frac{e^{ikx}}{\sqrt{l}} \sim \frac{\bar{g}_0}{\Delta} \cdot \sqrt{\frac{\pi v_g}{2l}}\, e^{ikx}$

$$J(\omega) = 2\pi \sum_k |g_k|^2 \delta(\omega - \omega_k) = \sum_k \underbrace{\left(\frac{2\pi}{l}\right)}_{\Delta k} \cdot v_g \pi \frac{1}{2} \left(\frac{\bar{g}_0}{\Delta}\right)^2 \delta(\omega - \omega_k) \cdot \omega_k$$

$$= \int_{\mathbb{R}} dk\, v_g \pi/2 \left(\frac{\bar{g}_0}{\Delta}\right)^2 \omega_k \delta(\omega - \omega_k) = \int d\omega\, \pi \left(\frac{\bar{g}_0}{\Delta}\right)^2 \omega_k \delta(\omega - \omega_k)$$

$$= \pi \alpha \omega^1 \qquad \alpha = \left(\frac{\bar{g}_0}{\Delta}\right)^2$$

Hence, if $\frac{g_0}{\Delta} \sim 10\%$ corresponds to breaking the RWA, in the continuum we would need $\frac{\gamma}{\Delta} \sim \left(\frac{\bar{g}_0}{\Delta}\right)^2 \sim 10\%$ which requires a much stronger coupling.

d) about $\bar{\xi}_0 \sim 0.33\,\Delta$

# Tuneable USC coupling

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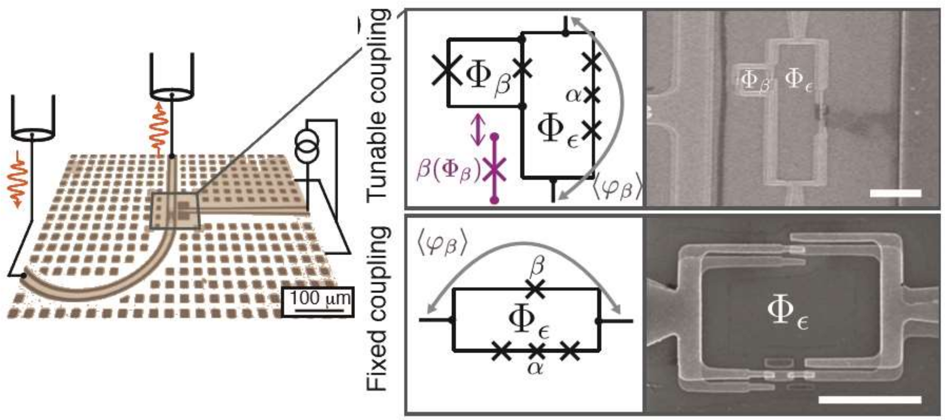
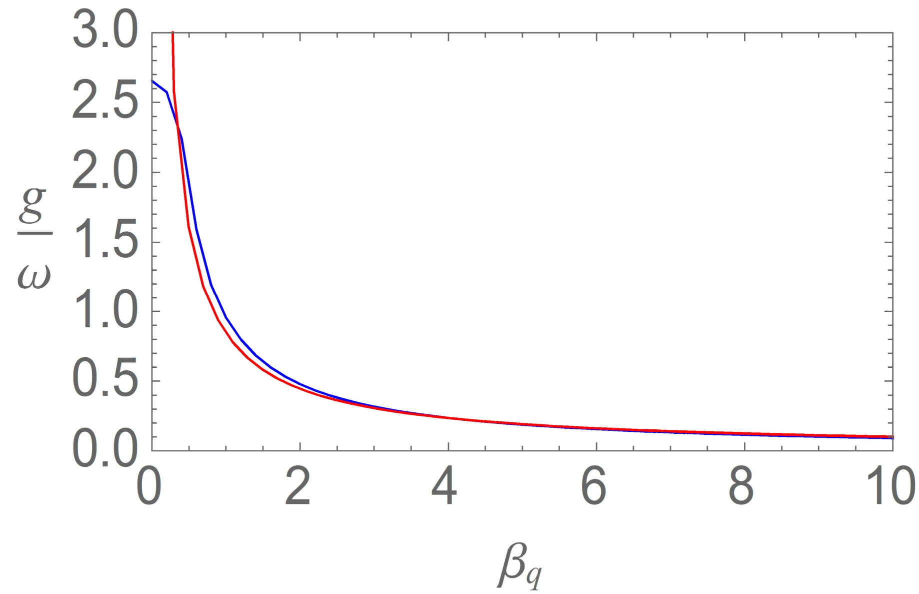
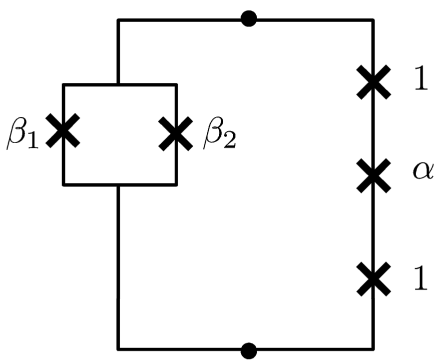
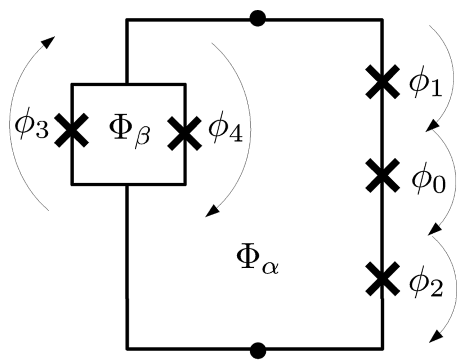


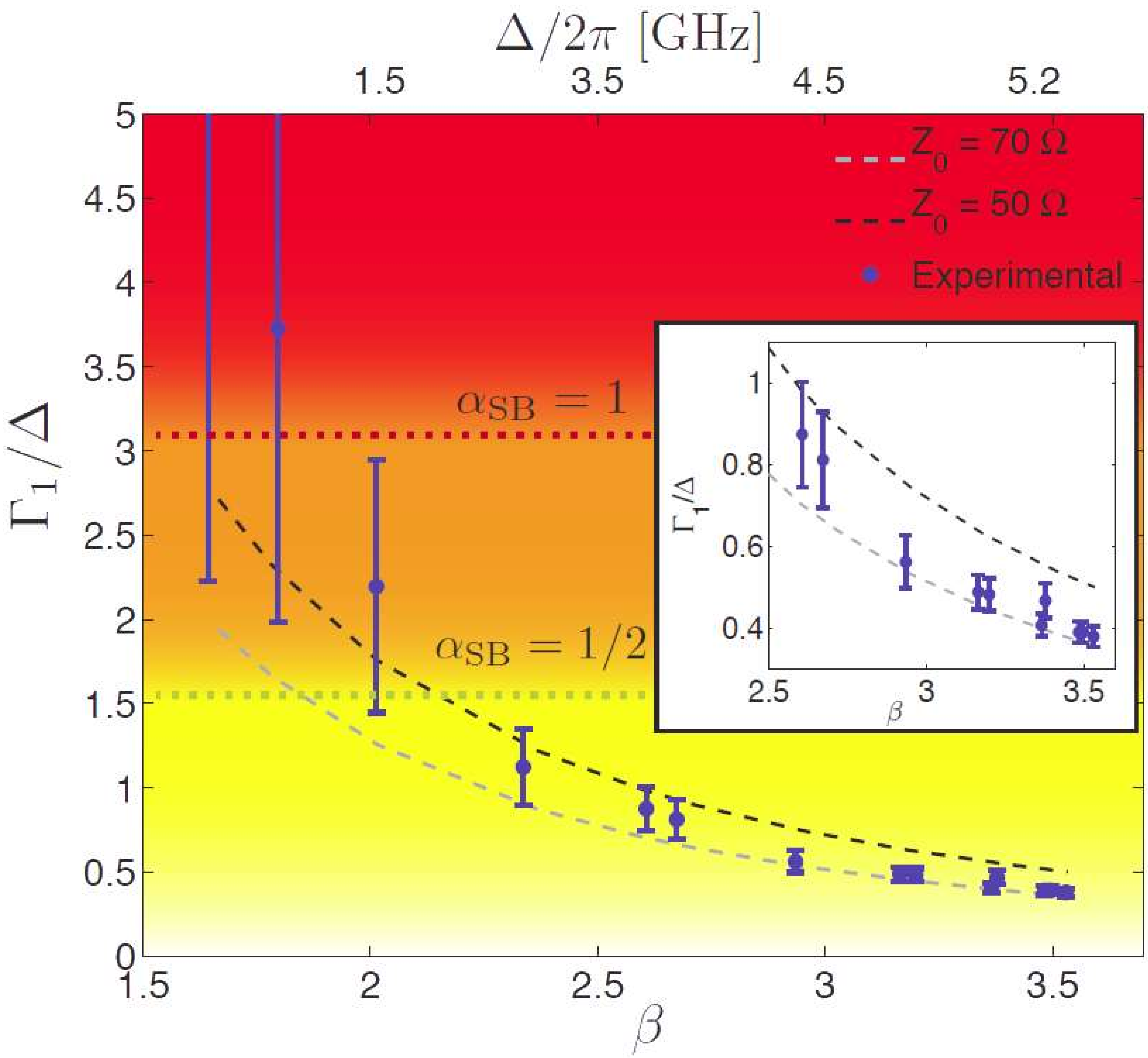




$$\phi_0 = \Phi_\alpha - \phi_1 - \phi_2 - \phi_3$$
$$\phi_4 = \Phi_\beta - \phi_3$$

# First experiments

Sunday, March 6, 2016 11:07 AM




*P. Forn-Díaz et al, arXiv:1602.00416*