Reminder

Wednesday, March 2, 2016 1:17 PM

Rabi model

$$H = \left(\frac{\Delta}{2}\sigma^z + \frac{\varepsilon}{2}\sigma^x\right) + \omega a^\dagger a$$
$$+ g\,\sigma^x (a^\dagger + a)$$

$\Downarrow$ RWA

Jaynes Cummings model

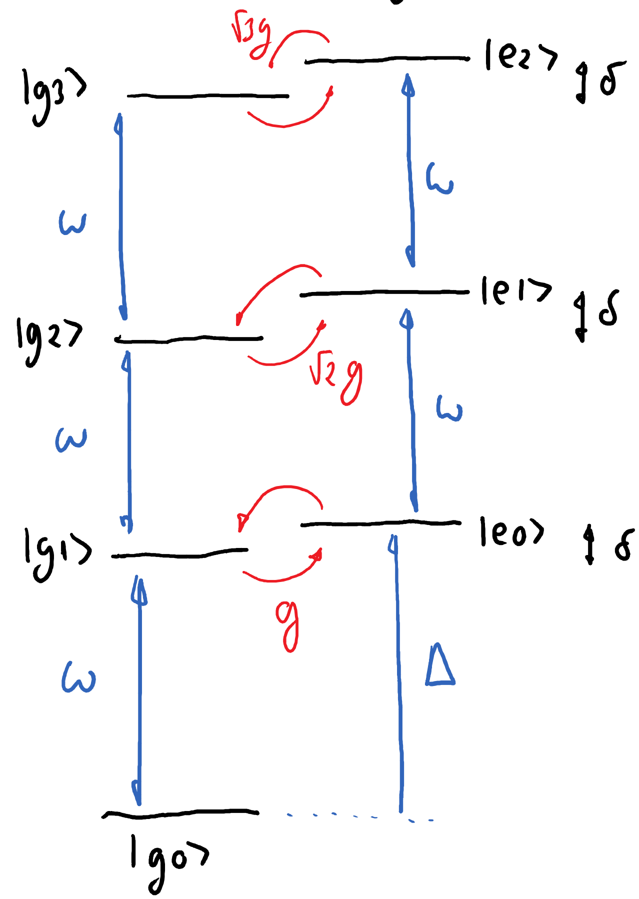
$$H = \left(\frac{\Delta}{2}\sigma^z + \frac{\varepsilon}{2}\sigma^x\right) + \omega a^\dagger a$$
$$+ g(\sigma^+ a + a^\dagger \sigma^-)$$

$\Downarrow$ dispersive

$$H = \frac{1}{2}\left(\Delta + \frac{g^2}{\delta}\right)\sigma^z$$
$$+ \left(\omega + \frac{g^2}{\delta}\sigma^z\right)a^\dagger a$$

$\delta = \Delta - \omega$

Jaynes-Cummings ladder



Toolbox

1) Coherent drive of the cavity

$|n\rangle \leftrightarrow |n \pm 1\rangle$

2) Coherent drive of the qubit

$|g\rangle \leftrightarrow |e\rangle$

3) Driving between different eigenstates of J.C. model

e.g $|g0\rangle \leftrightarrow |e0\rangle \pm |g1\rangle$

4) Rabi oscillations

$|gn\rangle \leftrightarrow |e\,n{-}1\rangle$

Photon number resolution

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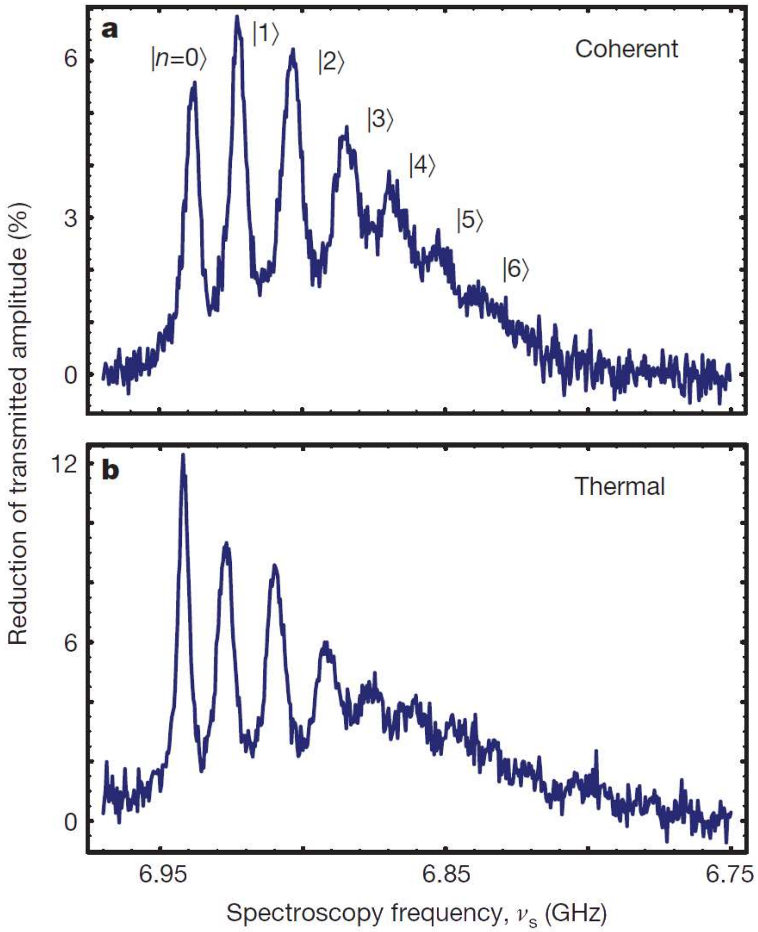

D.I. Schuster et al, Nature 445, 515 (2007)

The resonance frequency of the qubit is shifted by the photons

$$H = \frac{1}{2}\left(\Delta + \frac{g^2}{\delta} + \frac{2g^2}{\delta} a^\dagger a\right)\sigma^z + \omega a^\dagger a$$

Protocol:

1) Prepare cavity state
2) Drive the qubit $|g\rangle \leftrightarrow |e\rangle$ at different Ω
3) Drive the cavity at frequency ω and measure the phase shift to determine state of qubit σ^z

Sensitive to photon number distribution

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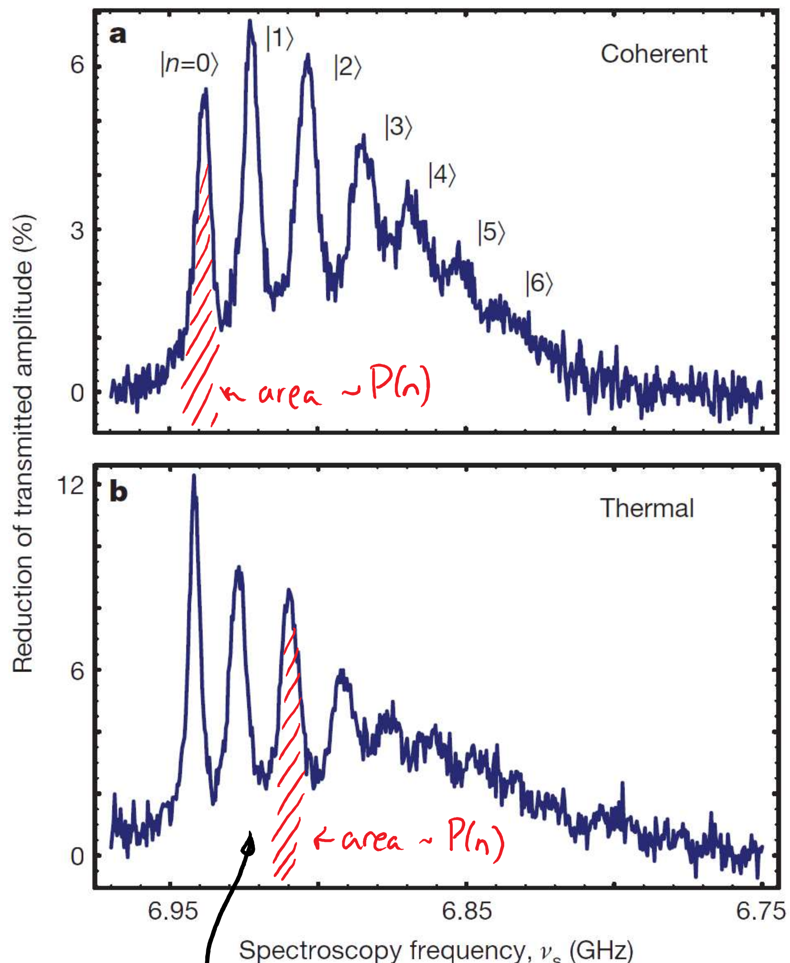
$$P(n) \sim \frac{e^{-\bar{n}}\,\bar{n}^{n}}{n!}$$

$\bar{n} = 3$

$$P(n) = \frac{\bar{n}^{n}}{(\bar{n}+1)^{n+1}}$$

$\bar{n} = 3$



- The measurement is proyective and once we start the dynamics the cavity ends up in one of the Fock states with some prob. $P(n) \sim |\langle n|\psi\rangle|^2$
- To get $P(n)$ we need many measurements
- Photons may be lost during the spectroscopy
  ↳ measurement error

created by Gaussian noise that reproduce T

Considerations

Wednesday, March 2, 2016 3:44 PM

- We need to be able to resolve transitions
  - $\delta \gg \kappa$ so that we can excite the qubit selectively
  - Total linewidth of n-th photon state

$$\gamma_n = \gamma/2 + \gamma_\phi + \frac{(\bar{n}+n)}{2}\kappa$$

intrinsic T_1 of qubit

intrinsic dephasing of qubit

decoherence induced by cavity losses

- Example: Schutz et al $\gamma \sim 2\pi \times 1\,\mathrm{MHz}$, $\gamma_\phi \sim 2\pi \times 1.8\,\mathrm{MHz}$, $\kappa \sim 2\pi \times 0{,}28\,\mathrm{MHz}$

$\delta \sim 2\pi \times 1.2\,\mathrm{GHz}$, $g = 2\pi \times 105\,\mathrm{MHz}$

$\chi = g^2/|\delta| = 2\pi \times 12\,\mathrm{MHz}$

Engineering of photon Fock states

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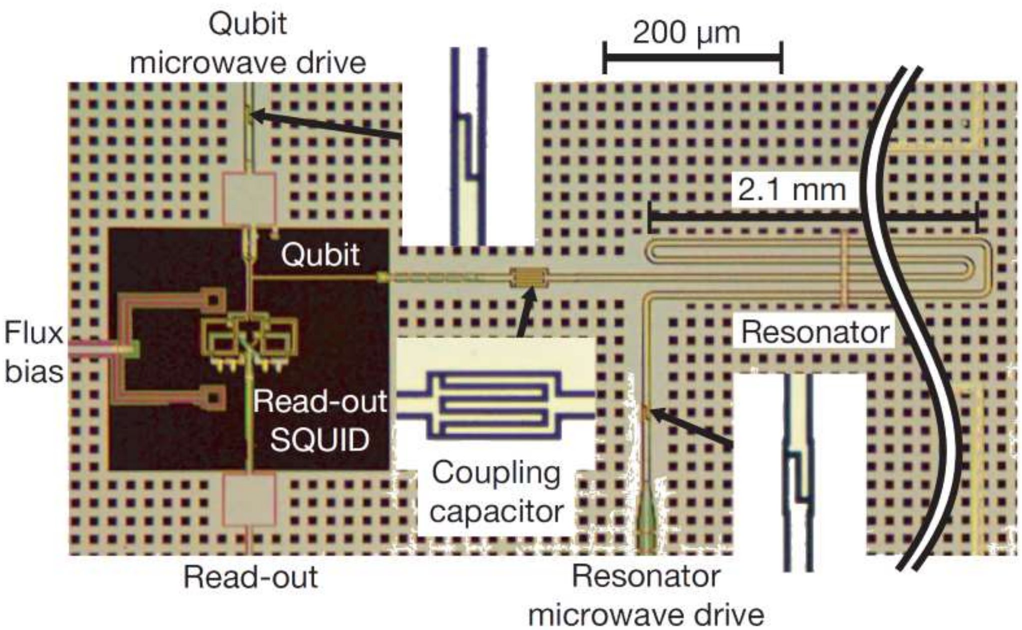
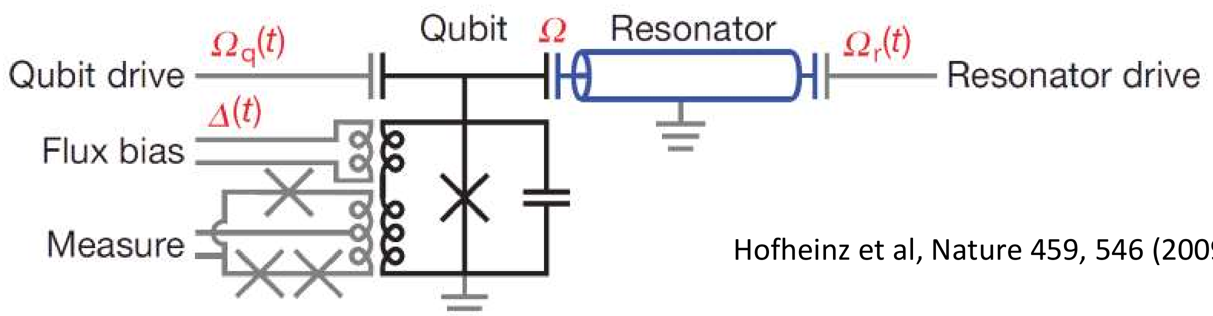


Hofheinz et al, Nature 454, 310 (2008)



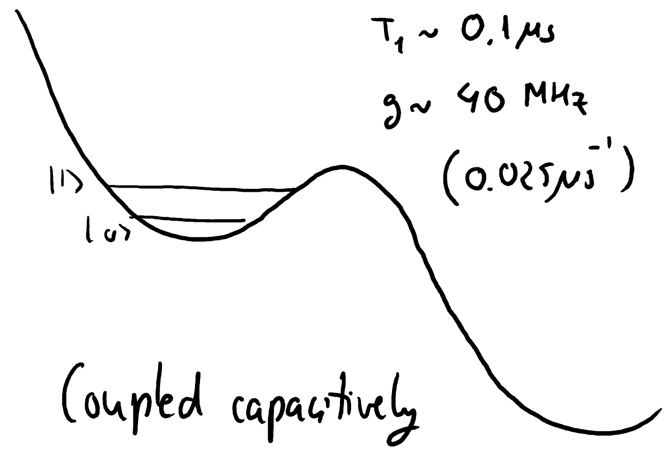
Hofheinz et al, Nature 459, 546 (2009)

Phase qubit

$T_2 \sim 0.5\mu s$

$T_1 \sim 0.1 \mu s$

$g \sim 40$ MHz $(0.025 \mu s^{-1})$



Coupled capacitively although inductive coupling would have also been possible

Engineering of photon Fock states (2)

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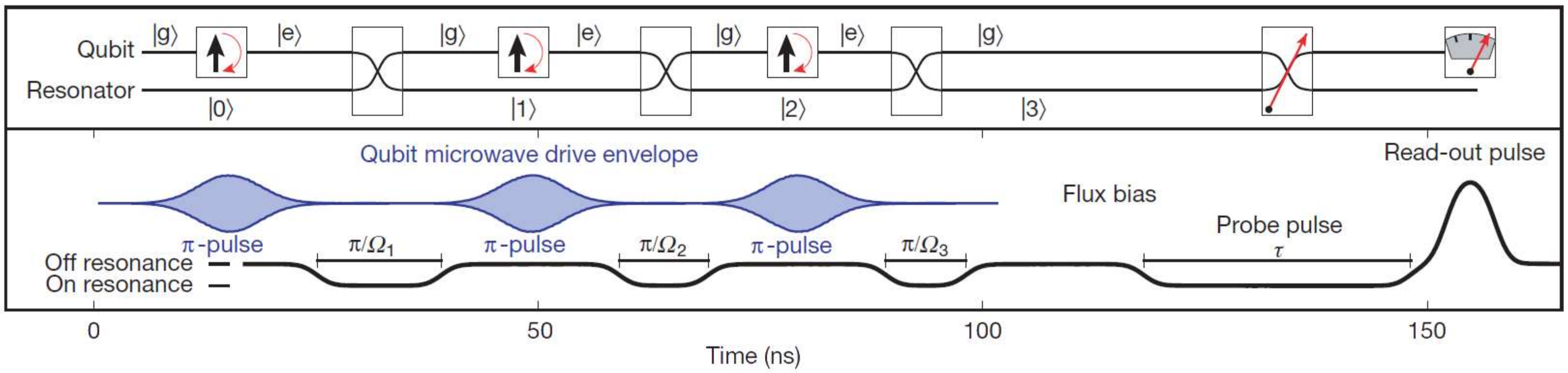
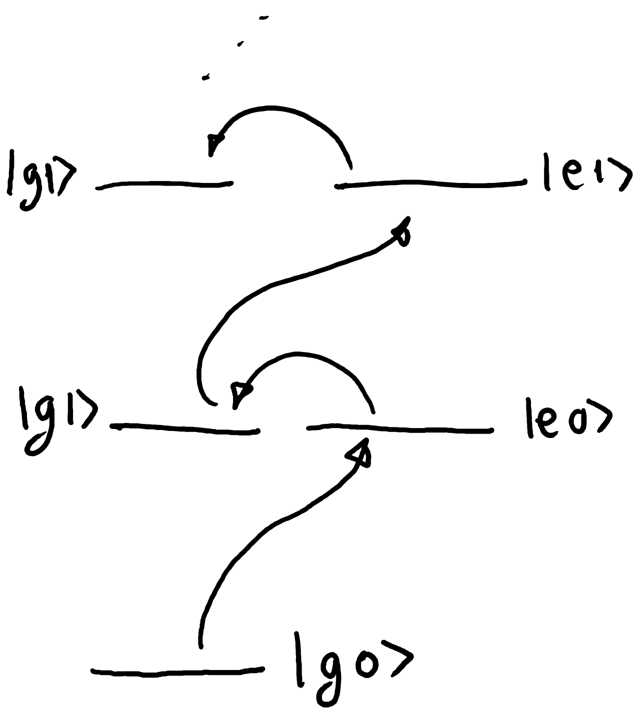


Hofheinz et al, Nature 454, 310 (2008)



* Protocol :

for $n = 1 \ldots N$

flip qubit state $|g\rangle \rightarrow |e\rangle$

wait for a time $\frac{2\pi}{\sqrt{n}\, g}$ to get $|gn\rangle$

end

Resonant measurement

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Assume $|\psi(0)\rangle = \left(\sum_n \psi_n |n\rangle\right) \otimes |g\rangle$

Bring qubit in resonance w. cavity :

$$H = \omega\left[\frac{1}{2}\sigma^z + a^\dagger a\right] + g\left(\sigma^+ a + a^\dagger \sigma^-\right)$$

$$= \left(-\frac{\omega}{2}\right) + \omega n\left[|gn\rangle\langle gn| + |e\,n\text{-}1\rangle\langle e\,n\text{-}1|\right] + \sqrt{n}\,g\left(|gn\rangle\langle e\,n\text{-}1| + h.c.\right)$$

Thus $|\psi(t)\rangle = \sum_n \psi_n \left[\cos\left[\sqrt{n}\,g\,t\right]|gn\rangle + i\sin\left(\sqrt{n}\,g\,t\right)|e\,n\text{-}1\rangle\right]e^{-i\omega n t}$

$$P_e(t) = \sum_n \sin^2\left(\sqrt{n}\,g\,t\right)|\psi_n|^2 \sim 1 - \frac{1}{2}\sum_n \cos\left(2\sqrt{n}\,g\,t\right)|\psi_n|^2$$

We can Fourier transform $P_e(t)$ $\begin{cases} \text{for one realization and get "}n\text{"} \\ \text{averaged over (3000) expts and get } |\psi_n|^2 \ \forall n \end{cases}$

Resonant measurement (2)

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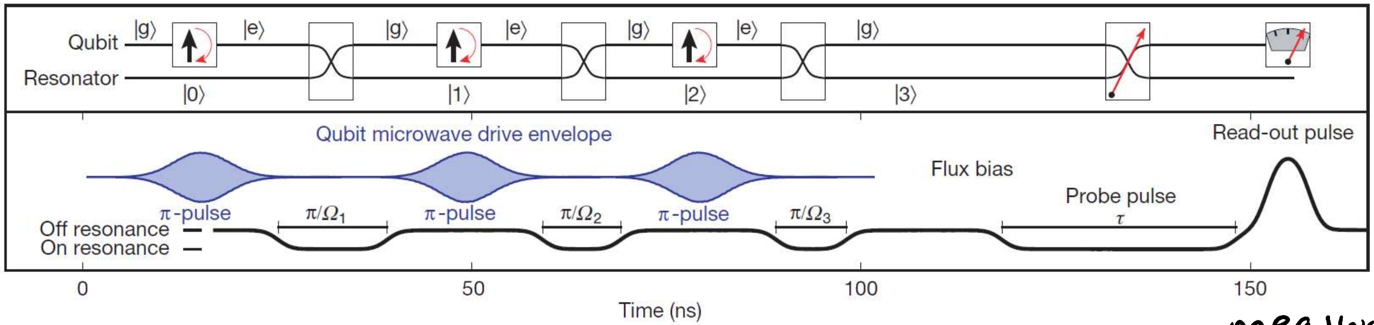
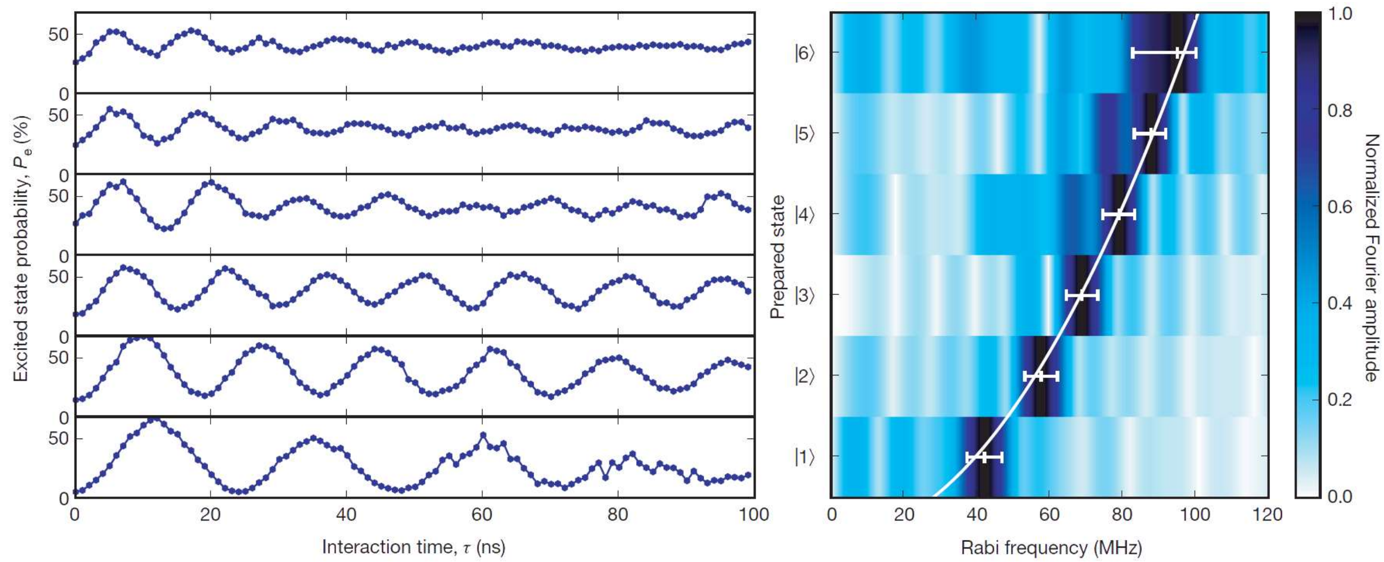


measurement traces



Qubit and cavity are brought into resonance.

From $P_e(t)$ we gather all photon components

Hofheinz et al, Nature 454, 310 (2008)

Engineering of superpositions

C K Law & J Eberly
PRL 76 1055 '96

We want to create an arbitrary state

$$|\psi(T)\rangle = \sum_n \psi_n |n\rangle$$

We will show that it can be done with two operations

$$S_n \approx \exp\left(-i\, t_n g \left(\sigma^+ a + a^+ \sigma^-\right)\right) \quad \sim \quad |e\, n-1\rangle \leftrightarrow |g\, n\rangle$$

$$Z_n \approx \exp\left(-i\, \phi_n |e\rangle\langle e|\right) \quad \sim \quad \text{phase on } |e\rangle \text{ w.r.t. } |g\rangle$$

$$Q_n \approx \exp\left(-i\, \tau_n \left(\Omega e^{i\theta_n/2} \sigma^+ + h.c.\right)\right) \quad \sim \quad |e\, n\rangle \leftrightarrow |g\, n\rangle$$

so that

$$|\psi(T)\rangle = S_N Q_N \dots S_1 Q_1 |g, 0\rangle$$

$$T = \sum_n (t_n + \tau_n) = \text{total time}$$

Derivation of protocol

Thursday, March 3, 2016 8:23 AM

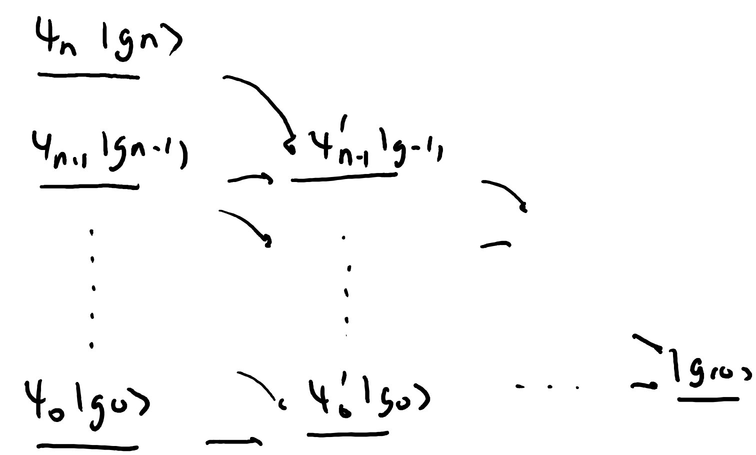
Solve for $|g,0\rangle = Q_1^+ Z_1^+ S_1^+ \dots Q_N^+ Z_N^+ S_N^+ |\psi\rangle$

We write $|F_M\rangle = Q_M^+ Z_M^+ S_M^+ |F_{M+1}\rangle$

$\alpha_j = \langle g,j | F_{j+1}\rangle$, $\beta_j = \langle e, j-1 | F_{j+1}\rangle$, $\mu_j = \langle e, j-1 | S_j^+ | F_{j+1}\rangle$, $\nu_j = \langle e, j-1 | S_j^+ | F_{j+1}\rangle$

$$\alpha_j \cos(g\sqrt{j}\, t_j) + i\beta_j e^{-i\phi_j} \sin(g\sqrt{j}\, t_j) = 0$$

$$\mu_j \cos(\Omega_j \tau_j) + i\nu_j e^{i\theta_j} \sin(\Omega \tau_j) = 0$$



These equations can be solved always and they amount to reconstructing the state from the initial distribution $|\psi\rangle$, moving population from $|j\rangle \to |j-1\rangle$ on each step

Example

Thursday, March 3, 2016 8:35 AM

Table 1 | Sequence to generate the resonator state $|\psi\rangle = |1\rangle + i|3\rangle$

Sequence of states, operations	Operational parameter	System state, parameter value
$\|\psi\rangle$		$\|g\rangle(0.707\|1\rangle + 0.707i\|3\rangle)$
S_3	$\tau_3\Omega$	1.81
Q_3	q_3	3.14
$\|\psi_2\rangle$		$\|g\rangle(-0.557i\|0\rangle + 0.707\|2\rangle) + 0.436\|e\rangle\|1\rangle$
Z_2	$t_2\Delta$	4.71
S_2	$\tau_2\Omega$	1.44
Q_2	q_2	$-2.09 - 2.34i$
$\|\psi_1\rangle$		$(0.553 - 0.62i)\|g\rangle\|1\rangle - (0.371 + 0.416i)\|e\rangle\|0\rangle$
Z_1	$t_1\Delta$	3.26
S_1	$\tau_1\Omega$	1.96
Q_1	q_1	$-2.71 - 1.59i$
$\|\psi_0\rangle$		$(0.197 - 0.98i)\|g\rangle\|0\rangle$

← target state

← first state with $n < 3$ photons

← second state, now $n < 2$

← finally $n = 0$

order in which gates are applied experimentally

Hofheinz et al, Nature 454, 310 (2008)

State reconstruction and tomography

Thursday, March 3, 2016 8:42 AM

Problem: 1) We create distributions $|\psi(\tau)\rangle = \left(\sum_n \psi_n |n\rangle\right) \otimes |g\rangle$

of which we can measure $|\psi_n|^2$, but not yet $\arg(\psi_n)$

2) More generally, our states will <u>not</u> be pure but be density matrices

$$\rho \approx \left(\sum_{nm} \rho_{nm} |n\rangle\langle m|\right) \otimes |g\rangle\langle g| + \rho' \longrightarrow \text{preparation error}$$

also prep. errors because $\rho_{nm} = \psi_n \psi_m^* + \text{errors}$

out of which we can only reconstruct diagonal elts

$$\rho_{nn} = \mathrm{tr}\left(|n\rangle\langle n| \otimes |g\rangle\langle g| \, \rho\right)$$

Wigner Junction

Thursday, March 3, 2016 8:46 AM

$$H = \frac{1}{2}\hbar\omega(q^2+p^2) = \hbar\omega(a^\dagger a + 1/2), \quad p = \frac{1}{\sqrt{2}} i(a^\dagger - a), \quad q = \frac{1}{\sqrt{2}}(a + a^\dagger)$$

It's a quasiprobability distribution defined for arbitrary mixed or pure states

$$W(\alpha) = W\left(\frac{q}{2} + i\frac{p}{2}\right) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \langle q + x | \rho | q - x \rangle e^{ixp/2}\, dx$$

eigenstate of "x"

$$P(q) = \langle q | \rho | q \rangle = \int_{-\infty}^{\infty} W(q,p)\, dp \qquad P(p) = \langle p | \rho | p \rangle = \int_{-\infty}^{\infty} W(q,p)\, dq$$

eigenstate of position

eigenstates of "p"

It contains information about any expectation values of operators defined in terms of "x" and "p"

$$\langle \hat{G} \rangle = \int_{-\infty}^{+\infty} dq\, dp\; W(q,p)\; g(q,p)$$ where "g" is the expression of $\hat{G}$ in those variables

Wigner functions (2)

Thursday, March 3, 2016 11:21 AM

A Wigner function can be obtained as an expectation value over displaced states

$$W(\alpha) = \frac{2}{\pi} \, tr\left((-1)^{a^\dagger a} \rho(-\alpha) \right) = \frac{2}{\pi} \langle \Pi \rangle_{\rho(-\alpha)}$$

$$\rho(\alpha) = D(\alpha) \, \rho \, D(\alpha)^\dagger$$

$$D(\alpha) = \exp\left(\alpha a^\dagger - \alpha^* a \right) \rightarrow \text{displacement operator}$$

$$\Pi = (-1)^{a^\dagger a} = \sum_n (-1)^n |n\rangle\langle n| \rightarrow \text{parity operator}$$

The density matrix ρ can be recovered by fitting $W(\alpha)$ for a large number of displacements, assuming a cut off $\rho(0) = \sum_{n,m \leq N} \rho_{nm} |n\rangle\langle m|$

Paradigmatic states

Thursday, March 3, 2016 12:14 PM

a) Coherent states

$$W_\beta(\alpha) = \frac{2}{\pi} \exp\left(-2|\beta-\alpha|^2\right) \qquad W(q,p) = \frac{2}{\pi} e^{-\frac{1}{2}(q^2+p^2)}$$

b) Fock states

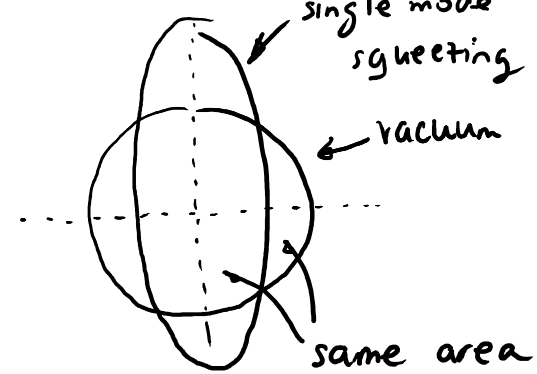
$$W_n(q,p) = \frac{2}{\pi}(-1)^n L_n\left(4(q^2+p^2)\right) e^{-2(p^2+q^2)}$$

← will have negative values for $n>0$

Laguerre polynomials

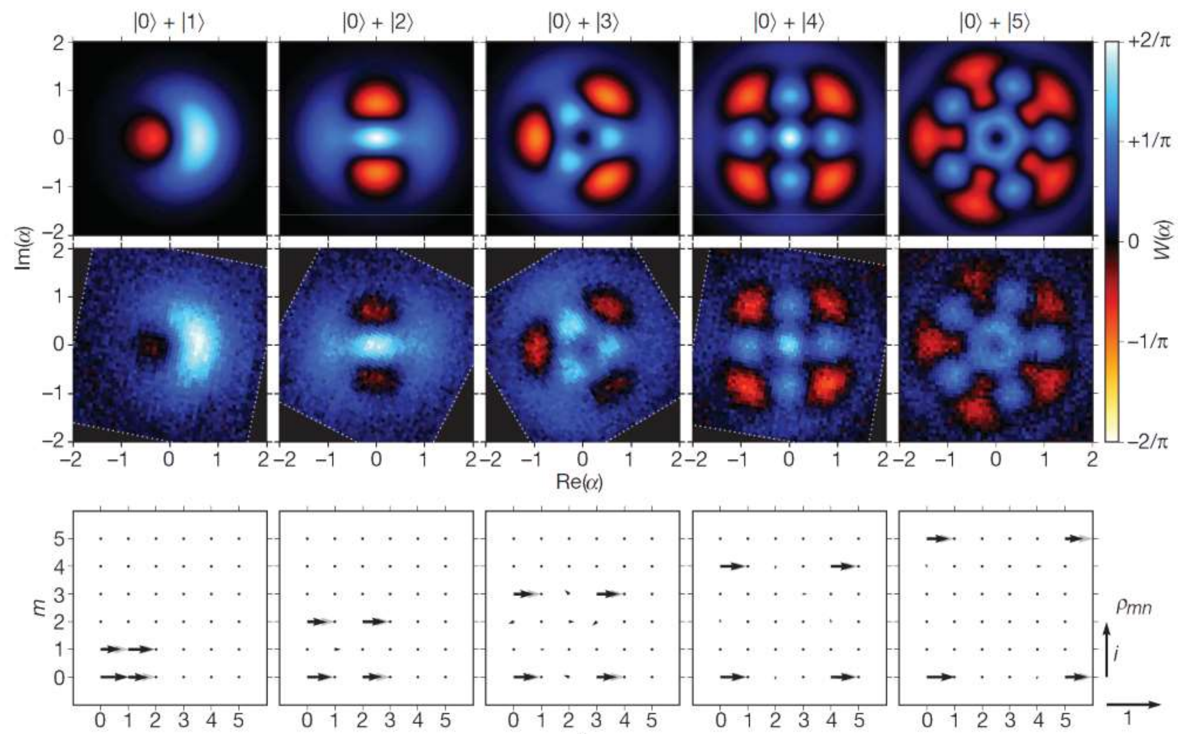
c) Squeezed states (vacuum)

$$W_r(q,p) = \frac{2}{\pi} \exp\left[-\frac{1}{2}\left(q^2 e^{-2r} + p^2 e^{+2r}\right)\right]$$



Experiment : statics

Thursday, March 3, 2016 11:34 AM



Hofheinz et al, Nature 454, 310 (2008)

Protocol:

For each "α"

- prepare $|\psi\rangle$
- displace cavity
- measure $\rho_{nn}(\alpha) = |\psi_n(\alpha)|^2$
- estimate $\langle \Pi \rangle$
- add up to $W(\alpha)$

Then reconstruct ρ using this info

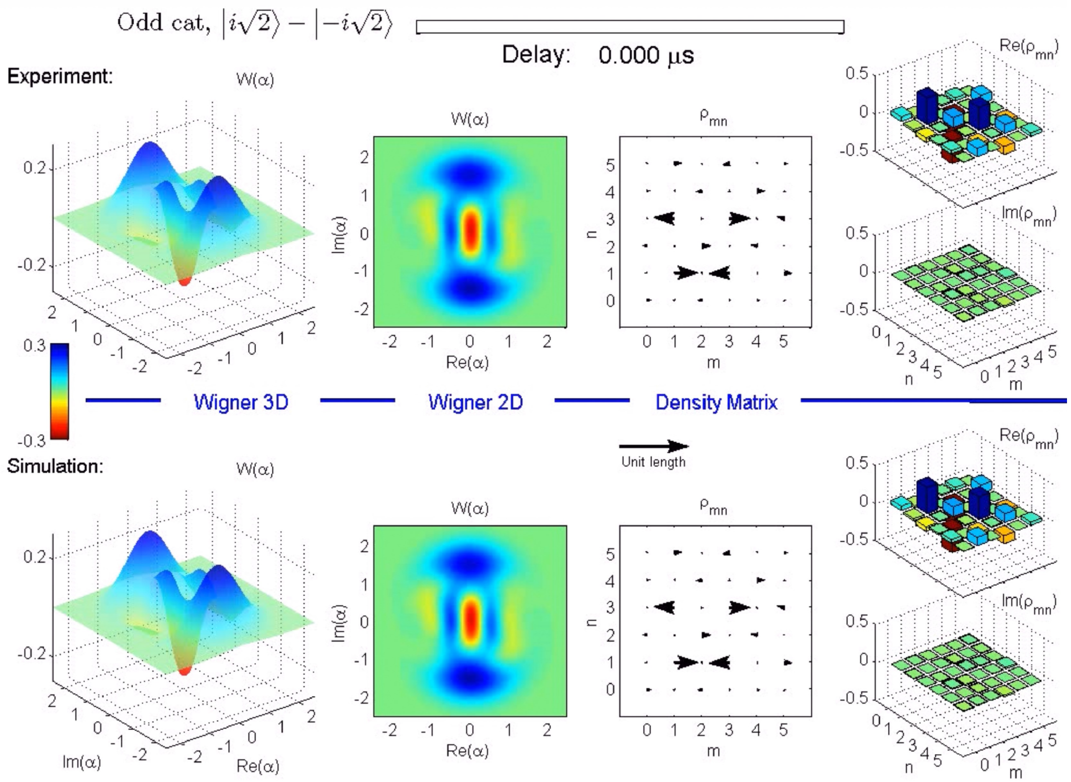
↳ maximum likelihood

↳ or eliminate 'wrong' eigenvalues

Compute new, smoothed $W(\alpha)$ from estimated ρ

Experiment : dynamics

Thursday, March 3, 2016 12:09 PM



http://web.physics.ucsb.edu/~martinisgroup/movies.shtml

Dynamics of Wigner function $W(\alpha,t)$

Quantum superpositions are destroyed by decoherence ($\rho_{nm} \to 0 \quad n \neq m$) and dissipation ($\rho_{nn} \to 0 \quad n > 0$)

This is most obvious for nonclassical states such as

a) Fock states $|n\rangle$

b) Schrödinger cats $|\alpha\rangle \pm |-\alpha\rangle$

because the Wigner function becomes nonnegative and the state therefore stops being nonclassical

Schrödinger cat states

Thursday, March 3, 2016 3:19 PM

A Schrödinger cat is a superposition state between two distinguishable configurations of a system.

- The two states $|\psi\rangle \sim \frac{1}{\sqrt{2}}|A\rangle + \frac{1}{\sqrt{2}}|B\rangle$ should ideally be orthogonal
- We expect $|A\rangle$ and $|B\rangle$ to be distinguishable by some observable, Θ
- A cat is "macroscopic" when it involves a large number of subsystems, (N particles, atoms, etc) and they conspire to make $\langle\Theta\rangle_A - \langle\Theta\rangle_B \propto \Theta(N)$

A typical example is a Schrödinger cat of $N = |\alpha|^2$ atoms

$$|\psi\rangle \sim \frac{1}{\sqrt{2}}\left(|\alpha\rangle + e^{i\theta}|-\alpha\rangle\right) \xrightarrow{\theta=0} \frac{1}{\sqrt{2}}\left(|\alpha\rangle + |-\alpha\rangle\right) = \sum_n e^{-|\alpha|^2/2} \frac{\alpha^{2n}}{\sqrt{(2n)!}}|2n\rangle \text{ even}$$

$$\xrightarrow{\theta=\pi} \frac{1}{\sqrt{2}}\left(|\alpha\rangle - |-\alpha\rangle\right) = \sum_n e^{-|\alpha|^2/2} \frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}}|2n+1\rangle \text{ odd}$$

Cavity displacement (reminder)

Thursday, March 3, 2016 8:44 PM

Take a cavity coupled to an external AC drive

$$H = \omega a^\dagger a + \Omega \cos(\omega_d t + \phi)(a + a^\dagger)$$

$D(-\beta)\, a\, D(\beta) = a + \beta$

$D(\beta) = \exp(\beta a^\dagger - \beta^* a)$

We can integrate Heisenberg dynamics of $a, a^\dagger$

$$\frac{d}{dt} a = -i[a, H] = -i\omega + \Omega \cos(\omega_d t + \phi)$$

$$a(t) = e^{-i\omega t} a(0) + e^{-i\omega t} \int_0^t e^{i\omega\tau} \Omega \cos(\omega_d t + \phi)\, d\tau$$

$$\sim e^{-i\omega t}\left[a(0) + t\Omega e^{-i\phi}\right] \quad \text{when } \omega = \omega_d$$

If time evolution is given by $a(t) = U^\dagger(t)\, a\, U(t)$ with $U(t) = \mathcal{T} e^{-i\int_0^t H(\tau) d\tau}$

then we have $U(t) = D(t\Omega e^{-i\phi})$ implements a displacement along $\Omega e^{i\phi}$

Fabrication

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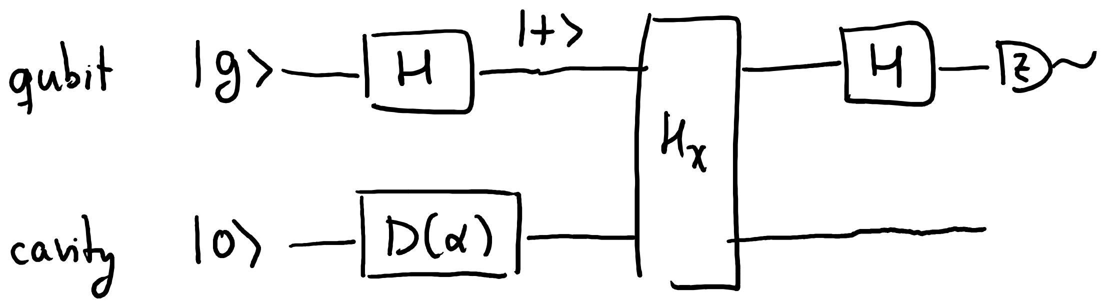
$$H_x = \left(\omega + \frac{g^2}{\delta}\sigma^z\right)a^\dagger a = \left(\omega + \chi\sigma^z\right)a^\dagger a$$



1) Create the state

$$|\psi_1\rangle = |+\rangle \otimes |\alpha\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle + |e\rangle\right) \otimes |\alpha\rangle$$

2) Decrease the detuning so that $\chi \cdot t = \frac{g^2}{\delta} t$ is large and wait for a time $T\chi \sim \pi/2$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|g\rangle |\alpha e^{-i(\omega-\chi)T}\rangle + |e\rangle |\alpha e^{-i(\omega+\chi)T}\rangle\right)$$

$$\approx \frac{1}{\sqrt{2}}\left(|g\rangle|\beta\rangle + |e\rangle|-\beta\rangle\right)$$

3) Apply a Hadamard again and measure σ^z: $|\psi_3\rangle = \frac{1}{2}|g\rangle\underbrace{\left(|\beta\rangle + |-\beta\rangle\right)}_{\psi_{even}} + \frac{1}{\sqrt{2}}|e\rangle\underbrace{\left(|-\beta\rangle - |\beta\rangle\right)}_{\psi_{odd}}$

Dissipation and decoherence

Thursday, March 3, 2016 3:27 PM

Macroscopic $N = |\alpha|^2 \gg$ cats decay or decohere faster

$$|n\rangle\langle n| \sim e^{-n\kappa t} |n\rangle\langle n| \quad \text{dissipation} \qquad \kappa \text{ cavity decay}$$

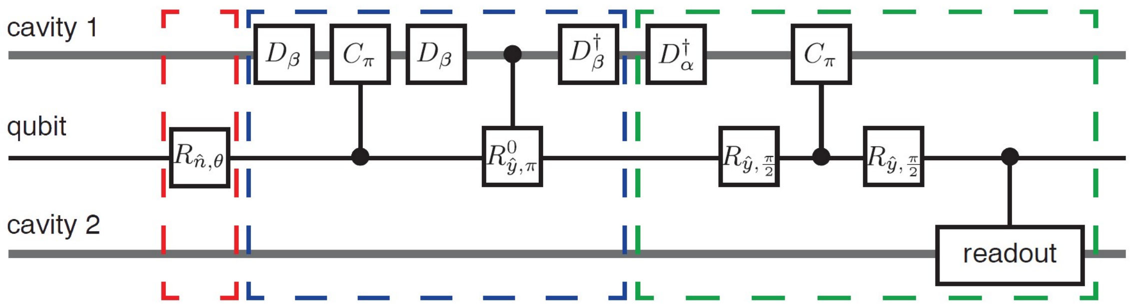
$$|n\rangle\langle m| \sim e^{-(n-m)\gamma_\phi t} |n\rangle\langle m| \quad \text{decoherence} \qquad \gamma_\phi = \frac{\kappa}{2} + \dots \quad \text{decoherence rate}$$

The first problem is purely classical but the second one is purely quantum and is related to the distinguishability of states $|A\rangle$ and $|B\rangle$ by any observable (such as $X = a + a^\dagger$, $P = i(a^\dagger - a)$) which can couple to the bath

Decoherence makes the creation and detection of large Schröedinger cats more challenging than simple qubit states superpositions.

Deterministic protocol

Thursday, March 3, 2016 3:42 PM



$D(\beta) = e^{\beta a^\dagger - \beta^* a}$

displacement operator

$C_\phi \sim e^{i\phi a^\dagger a |e\rangle\langle e|} + \mathbb{1} \otimes |g\rangle\langle g|$

$R^0_{y,\pi} \sim (|e\rangle\langle g| + |g\rangle\langle e|) \otimes |0\rangle\langle 0|$
$+ \mathbb{1} \otimes \sum_{n>0} |n\rangle\langle n|$

1) Prepare $|0\rangle \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$

2) Displace $|\beta\, g\rangle + |\beta\, e\rangle$

3) Conditional phase $H \sim \chi a^\dagger a |e\rangle\langle e|$ $\quad |\beta g\rangle + |-\beta e\rangle$ for $\chi T \sim \pi$

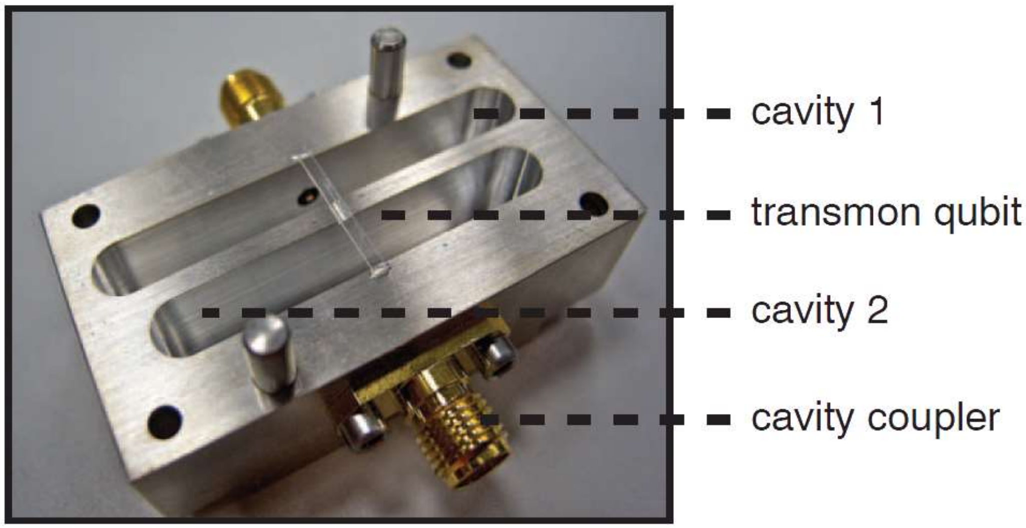
4) Displace $|0\, g\rangle + |-2\beta e\rangle$

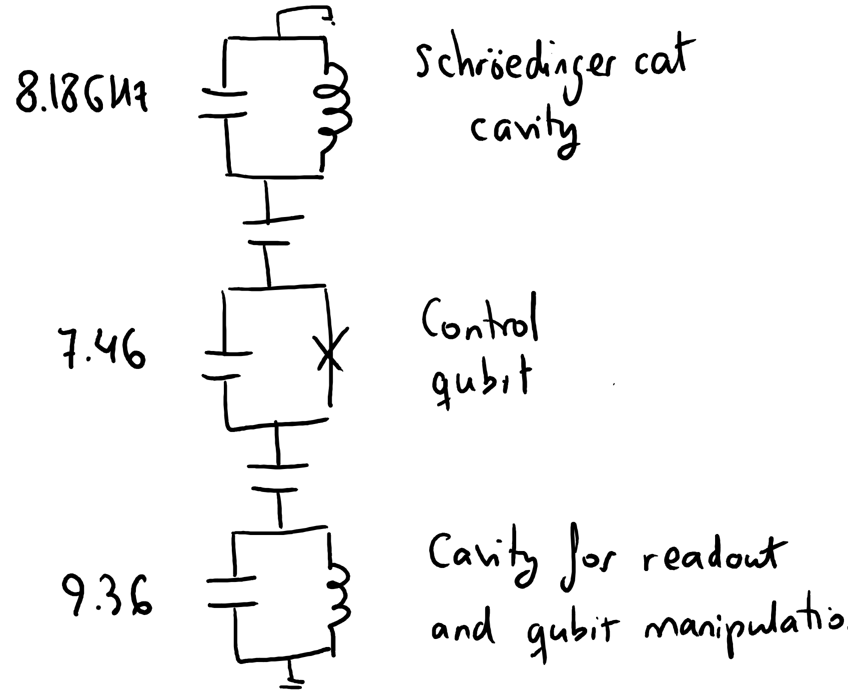
5) Conditional qubit rotation $\approx |0\, e\rangle + |-2\beta e\rangle$ provided $e^{-|\beta|^2} \approx 0$

6) Displace $(|\beta\rangle + |-\beta\rangle) \otimes |e\rangle$

Experiment

Thursday, March 3, 2016 6:00 PM





$\omega_{cav} = 2\pi \cdot 8.18$ GHz

$k_{cav} = 2\pi \cdot 7.2$ kHz

$\Delta_{qb} = 2\pi \cdot 7.46$ GHz

$\gamma = 2\pi \cdot 36$ kHz

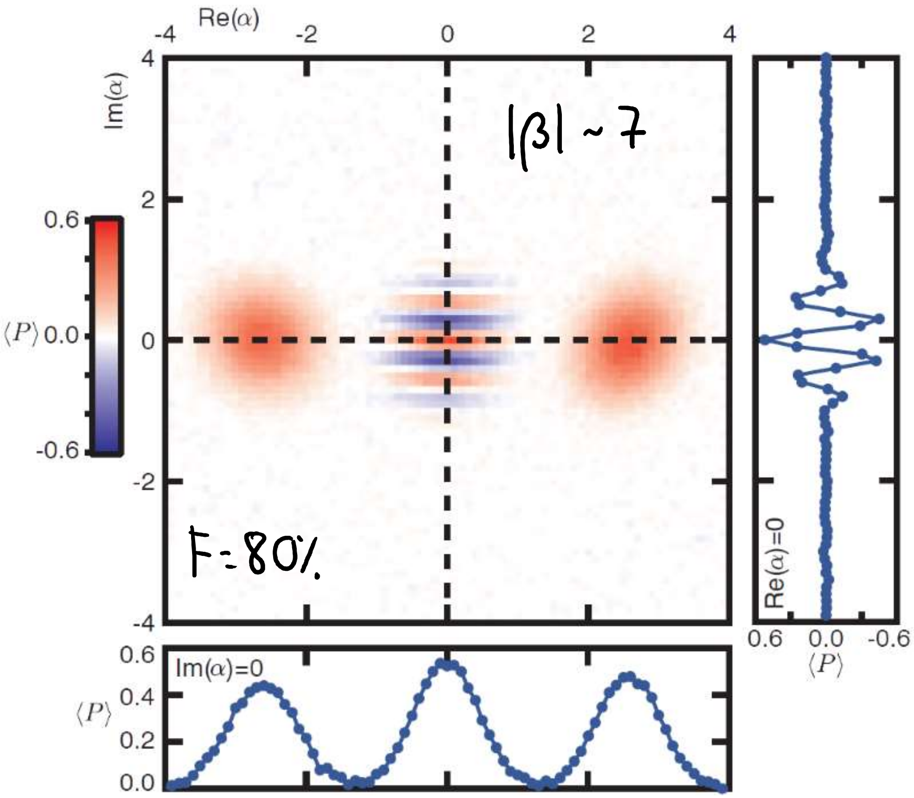
$\chi = 2.4$ MHz $\gg k, \gamma$ $\Rightarrow$ $T_\pi \sim \frac{1}{2.4} 10^{-6} = 400$ ns time for rotation

$T_1 \sim 4.4\,\mu s \gg T$

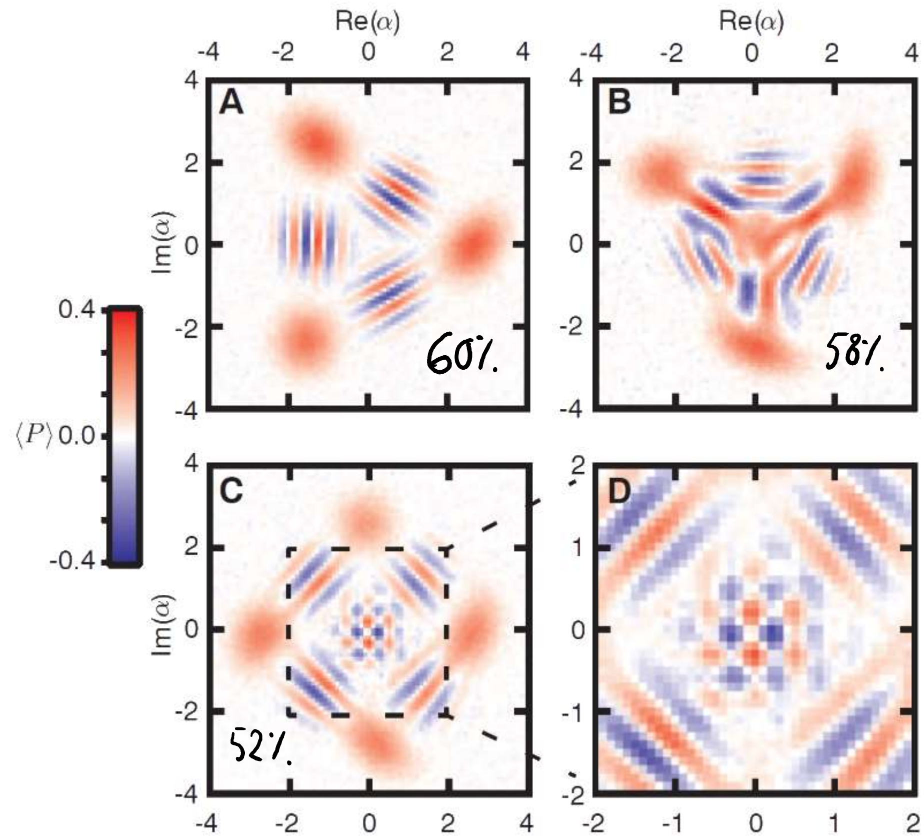
Examples

Thursday, March 3, 2016 4:00 PM

Note the interference fringes denoting non-classicality



$|\beta\rangle + |-\beta\rangle$



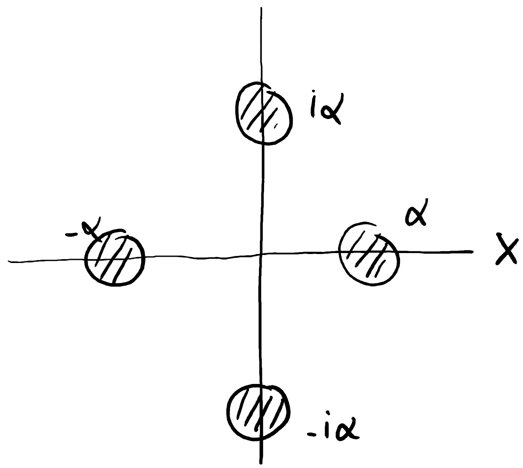
$\left(\sum_{n=1}^{N} |z_n\rangle \right) \frac{1}{\sqrt{N}}$

Bad effects: cavity kerr ⇒ loss of fidelity

Cat states as qubits

Thursday, March 3, 2016

We introduce a basis of four Schrödinger cats



$$|C_\alpha^\pm\rangle = \mathcal{N}\left(|\alpha\rangle \pm |-\alpha\rangle\right) \qquad \mathcal{N} = \left(2 + 2\,\mathrm{Re}\langle\alpha|-\alpha\rangle\right)^{1/2}$$

$$|C_{i\alpha}^\pm\rangle = \mathcal{N}\left(|i\alpha\rangle \pm |-i\alpha\rangle\right)$$

We use $|0\rangle = |C_\alpha^+\rangle$, $|1\rangle = |C_{i\alpha}^+\rangle$. The problem is that photon loss spoils these states

$$a|C_\alpha^+\rangle = \mathcal{N}\left[a|\alpha\rangle + a|-\alpha\rangle\right] = \alpha|C_\alpha^-\rangle$$

$$a|C_{i\alpha}^+\rangle = i\alpha|C_{i\alpha}^-\rangle$$

Encoding idea

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For each qubit state we find four states that are chained by photon losses

$$
\begin{aligned}
|\psi_\alpha^{(0)}\rangle &= \psi_g |C_\alpha^+\rangle + \psi_e |C_{i\alpha}^+\rangle \\
&\quad \big\downarrow \hat{a} \\
|\psi_\alpha^{(1)}\rangle &= \psi_g |C_\alpha^-\rangle + i\,\psi_e |C_{i\alpha}^-\rangle \\
&\quad \big\downarrow \hat{a} \\
|\psi_\alpha^{(2)}\rangle &= \psi_g |C_\alpha^+\rangle - \psi_e |C_{i\alpha}^+\rangle \\
&\quad \big\downarrow \hat{a} \\
|\psi_\alpha^{(3)}\rangle &= \psi_g |C_\alpha^-\rangle - i\,\psi_e |C_{i\alpha}^-\rangle
\end{aligned}
\qquad |\psi_\alpha^{(3)}\rangle \xrightarrow{\hat{a}} |\psi_\alpha^{(0)}\rangle
$$

We now need a simple measurement that tells us where we are in this chain. This measurement is the parity: by monitoring $\Pi = (-1)^{a^\dagger a}$ we know how many times we have moved away from $|\psi_\alpha^{(0)}\rangle$

Parity measurement

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$$|g\rangle|C_\alpha^\pm\rangle \xrightarrow{Had} \frac{1}{\sqrt{2}}\left(|g\rangle+|e\rangle\right)|C_\alpha^\pm\rangle$$

$$\text{dispersive} \rightarrow \frac{1}{\sqrt{2}}|g\rangle|C_\alpha^\pm\rangle + \frac{1}{\sqrt{2}}|e\rangle|C_{e^{i\chi t}\alpha}^\pm\rangle$$

$$|\alpha e^{i\pi}\rangle \pm |e^{i\pi}(-\alpha)\rangle$$

$$= |-\alpha\rangle \pm |\alpha\rangle = \pm\left(|\alpha\rangle \pm |-\alpha\rangle\right)$$

$$\chi t=\pi \rightarrow \frac{1}{\sqrt{2}}|g\rangle|C_\alpha^\pm\rangle \pm \frac{1}{\sqrt{2}}|e\rangle|C_\alpha^\pm\rangle$$

$$\xrightarrow{Had} \frac{1}{\sqrt{2}}\left(|g\rangle+|e\rangle\right)|C_\alpha^\pm\rangle \pm \frac{1}{\sqrt{2}}\left(|e\rangle-|g\rangle\right)|C_\alpha^\pm\rangle$$

$$= \begin{cases} |e\rangle|C_\alpha^+\rangle \\ |g\rangle|C_\alpha^-\rangle \end{cases}$$

photon state remains unaffected but we know whether parity changed

Encoding and decoding

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We can design four unitaries mapping qubit back and forth

$$(\psi_g|g\rangle + \psi_e|e\rangle)|0\rangle \xrightarrow{U_0} |g\rangle(\psi_g|C^+_\alpha\rangle + \psi_e|C^+_{i\alpha}\rangle) = |g\rangle|\psi^{(0)}_\alpha\rangle$$

$$\xrightarrow{U_1} |g\rangle(\psi_g|C^-_\alpha\rangle + i\psi_e|C^-_{i\alpha}\rangle) = |g\rangle|\psi^{(1)}_\alpha\rangle$$

$$\xrightarrow{U_2} |g\rangle(\psi_g|C^+_\alpha\rangle - \psi_e|C^+_{i\alpha}\rangle) = |g\rangle|\psi^{(2)}_\alpha\rangle$$

$$\xrightarrow{U_3} |g\rangle(\psi_g|C^-_\alpha\rangle - i\psi_e|C^-_{i\alpha}\rangle) = |g\rangle|\psi^{(3)}_\alpha\rangle$$

Note that only U_0 and U_1 are "different". The other ones can be made from $U_{0,1}$ times operations on the qubit.

Code space map

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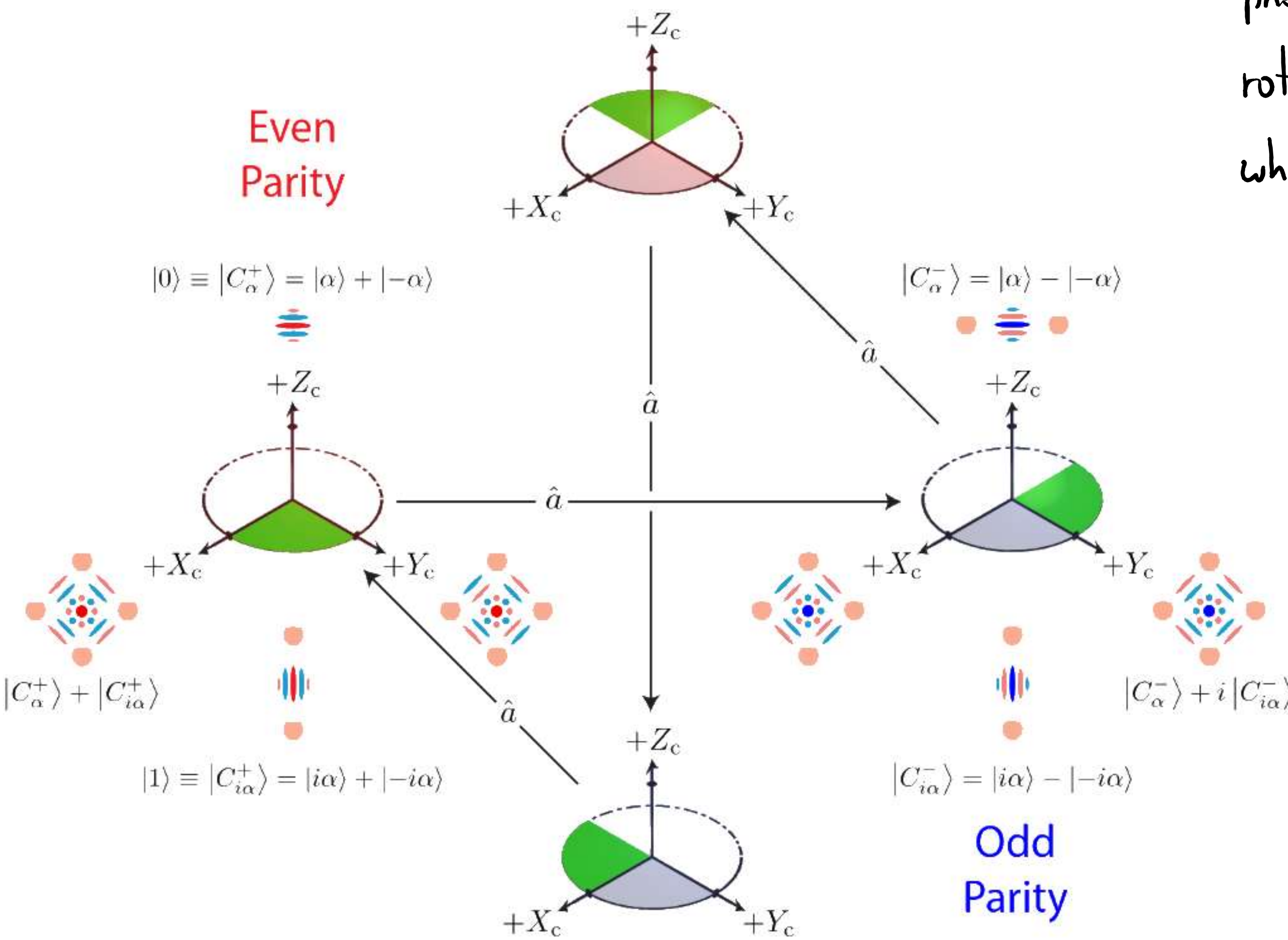
Chen Wang, Yvonne Y. Gao, Philip Reinhold, R. W. Heeres, Nissim Ofek, Kevin Chou, Christopher Axline, Matthew Reagor, Jacob Blumoff, K. M. Sliwa, L. Frunzio, S. M. Girvin, Liang Jiang, M. Mirrahimi, M. H. Devoret, R. J. Schoelkopf, arXiv:1601.05505



In this error correction code, photon loss amounts to rotations around "z_c" axis which may be undone

There are other errors such as kerr dephasing due to $H \sim \varepsilon\, a^\dagger a^\dagger a a$ photon nonlinearities that have to be compensated

Lifetimes

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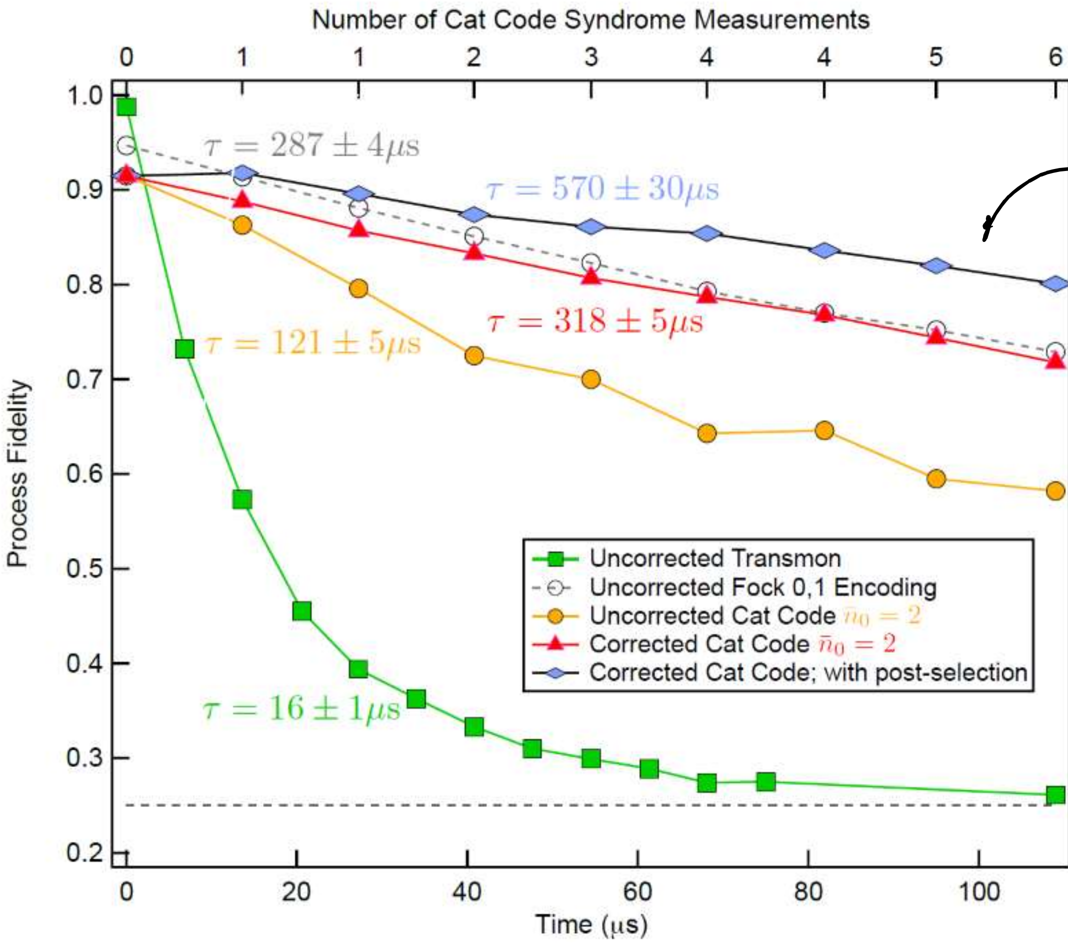

- Error is dominated by the ancilla
- It is possible to post-select quantum trajectories with high fidelity achieving longer survival rates
- Note that all the time the cavity loses energy

$$|C_\alpha^\pm\rangle \rightarrow |C_{\alpha e^{-kt/2}}^\pm\rangle$$

but this does not spoil information and can be corrected by pumping photons