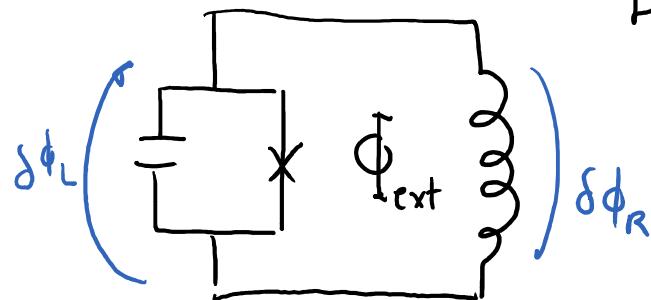


# Current states

lunes, 22 de febrero de 2016 9:18

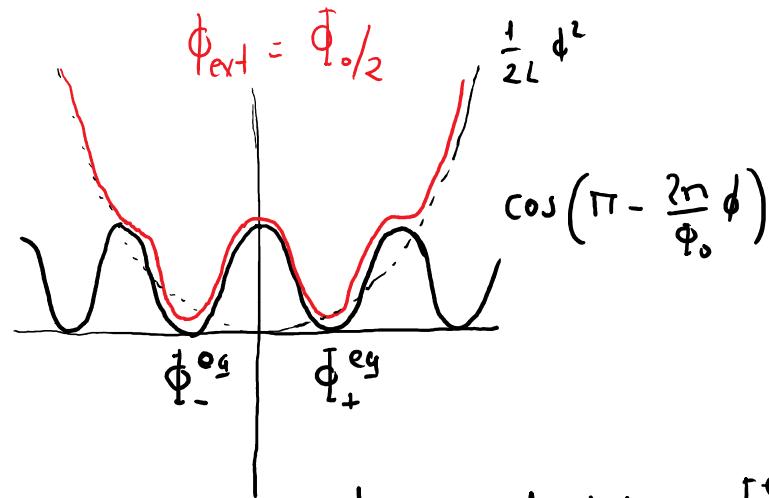
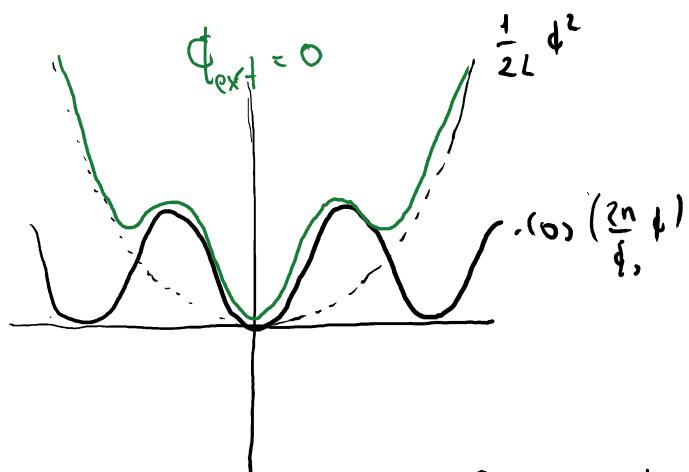


$$\delta\phi_L + \delta\phi_R = \Phi_{ext} + 2\pi m/\phi_0$$

Hamiltonian  $H \sim \frac{1}{2C} q^2 + \frac{1}{2L} (\phi)^2 - E_J \cos \left[ \frac{2\pi}{\phi_0} (\Phi_{ext} - \phi) \right]$

When  $\Phi_{ext} = 0$ , the minimum energy is obtained for  $\phi \approx 0$ , no flux difference

$$I(\Phi_{ext}=0) = I_c \sin \left( \frac{2\pi}{\phi_0} \cdot (\phi=0) \right) = 0$$



But when  $\Phi_{ext} \sim \frac{\phi_0}{2}$  we have two quasi-degenerate ground states  $\phi_{\pm}^{eq}$

### 3 junction flux qubit

lunes, 22 de febrero de 2016 9:26

A similar design, but we do not rely on the small inductance  $L$ .

$$\varphi_\alpha \left( \alpha E_J \times \Phi_{\text{ext}} \right) \varphi_1 \varphi_2 \quad \varphi_\alpha = \Phi_{\text{ext}} \frac{2\pi}{\phi_0} - \varphi_1 - \varphi_2 \sim \pi - (\varphi_1 + \varphi_2)$$

Working in the limit of one smaller junction  $\alpha \sim (0.7-0.9)E_J$ , the inductive energy reads

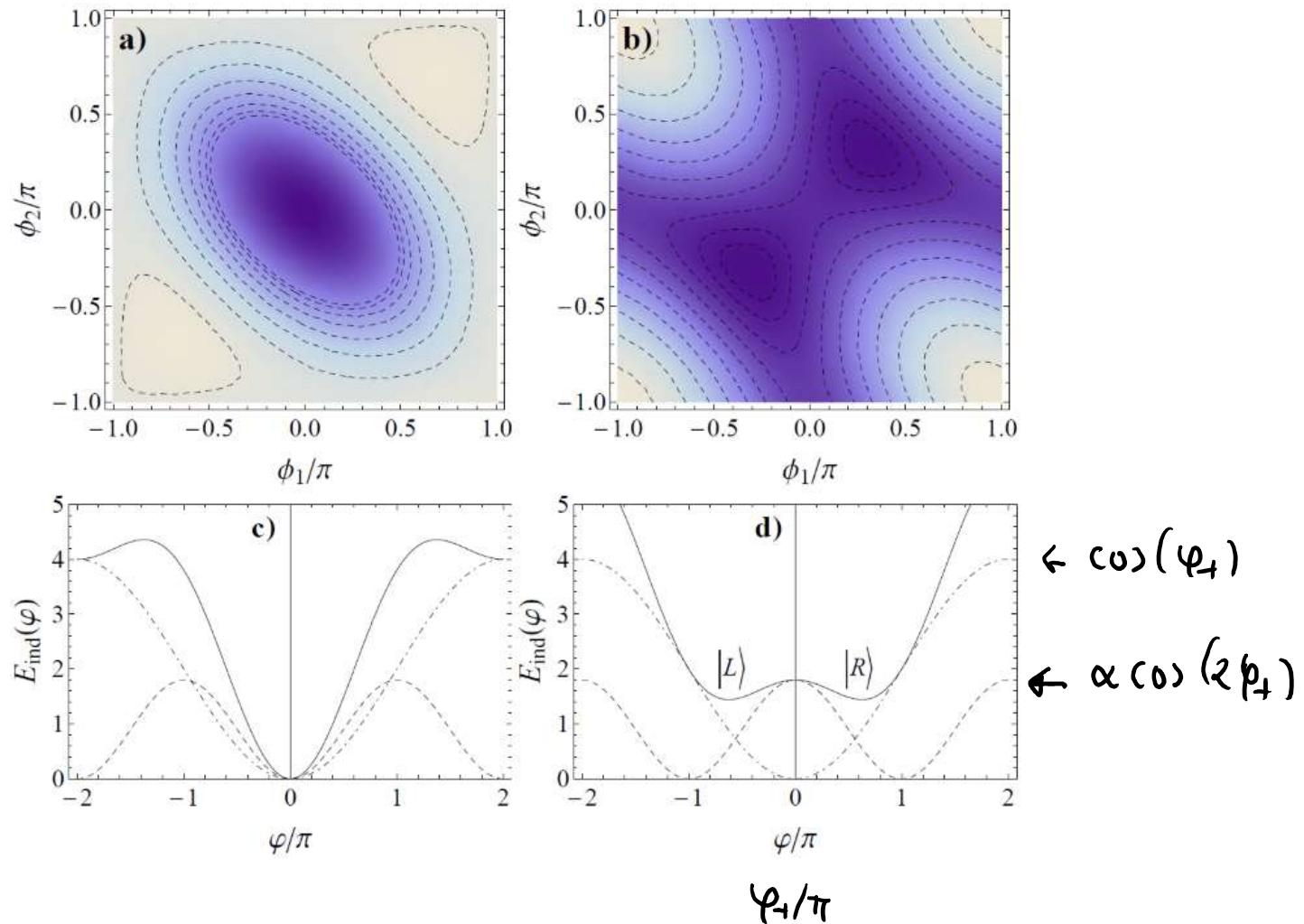
$$E_J \left[ \underbrace{\cos(\varphi_1) + \cos(\varphi_2)}_{2 \cos\left(\frac{\varphi_1 + \varphi_2}{2}\right)} + \alpha \cos(\pi - (\varphi_1 + \varphi_2)) \right]$$
$$2 \cos\left(\frac{\varphi_1 + \varphi_2}{2}\right) \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) - \alpha \cos(\varphi_1 + \varphi_2)$$

The minima are located along  $\varphi_1 = \varphi_2$ ,

$$E_J \left( 2 \cos(\varphi_1) - \alpha \cos(2\varphi_1) \right)$$

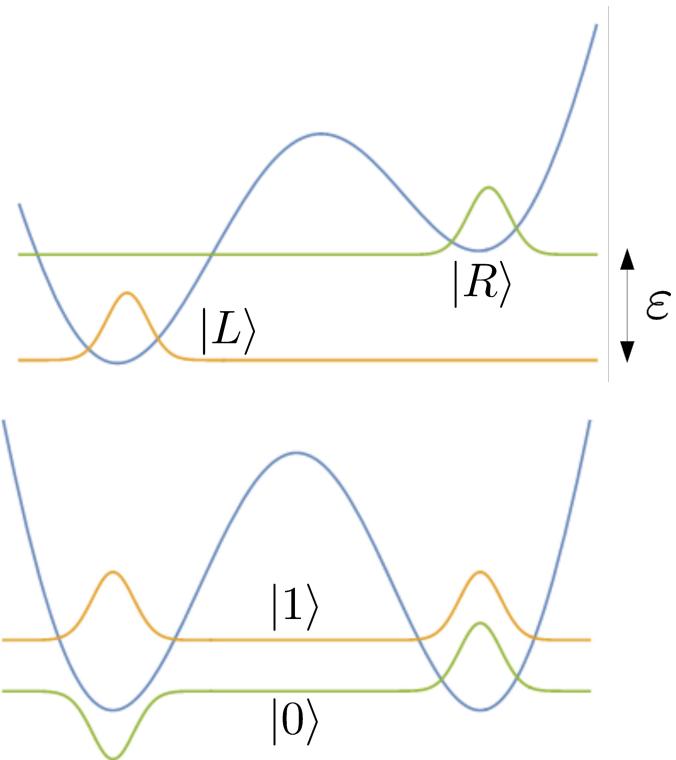
# Degenerate ground states

lunes, 22 de febrero de 2016 9:52



# Effective Hamiltonian

Lunes 12 de febrero de 2016 9:53



$$H = \frac{\Delta}{2} (|L\rangle\langle R| + |R\rangle\langle L|) + \epsilon (|R\rangle\langle R| - |L\rangle\langle L|)$$

$\Delta \propto E_J \times \text{overlap bw. } |L\rangle \text{ and } |R\rangle$

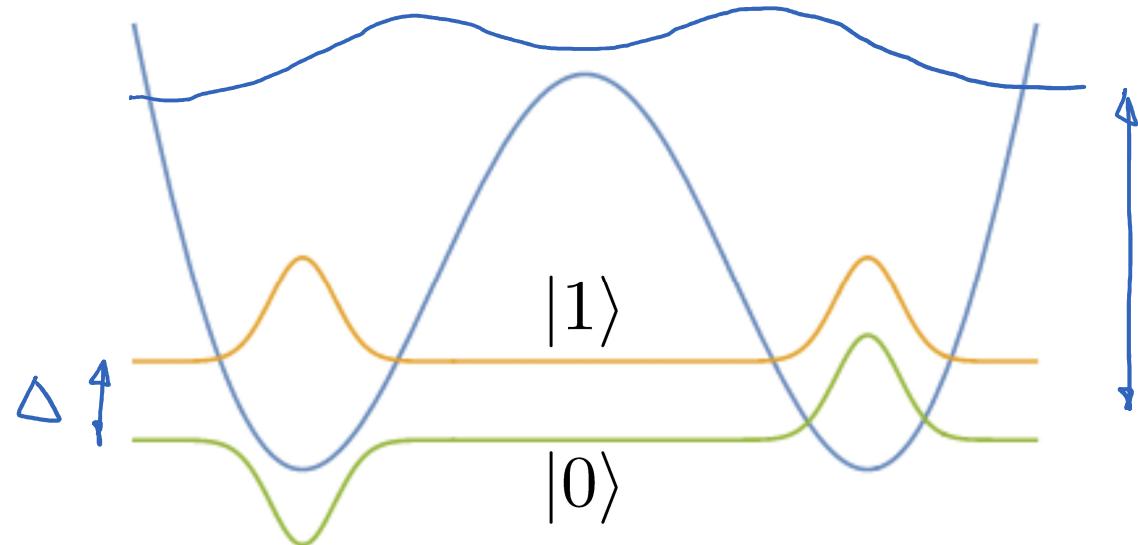
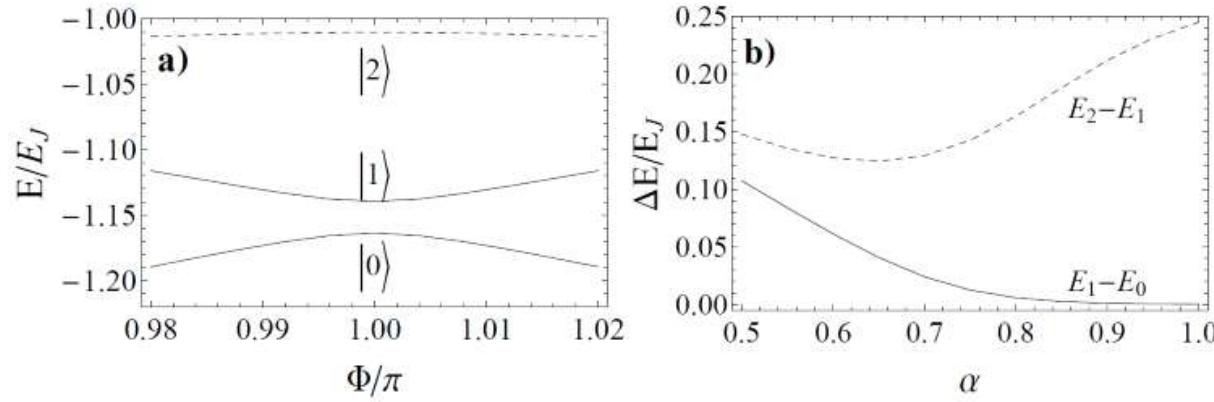
$$\epsilon \sim \frac{2\pi}{\Phi_0} (\Phi_{\text{ext}} - \Phi_0/2)$$

The two minima are connected due to quantum tunneling,  $\Delta$ . This amplitude decreases exponentially with  $E_J/E_C \Rightarrow$  very hard to "tune" or design

In addition to this, changes in the magnetic flux act as an effective magnetic field on the dipole  $|L\rangle\langle L| - |R\rangle\langle R|$

# Anharmonicity

lunes, 22 de febrero de 2016 10:09



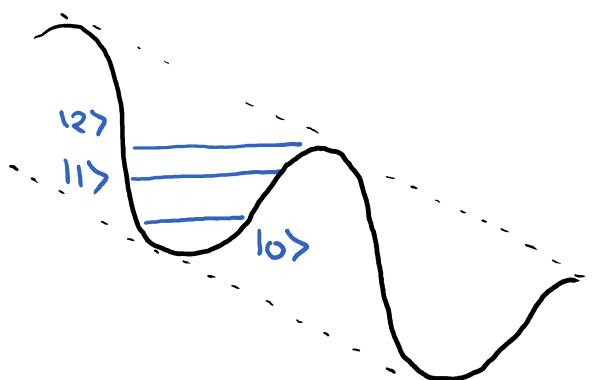
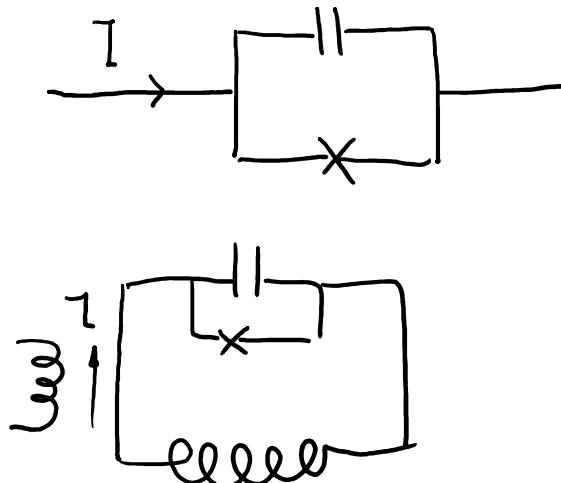
- \* The qubit arises from the splitting of one level in a ladder of broadly separated energy states
- \*  $\omega_{01} \ll \omega_{12}$  by one order of magnitude

$$\sim \sqrt{E_J E_C} \text{ anharmonic oscillator}$$

# Phase qubit

jueves, 25 de febrero de 2016 11:21

- \* A variation of the current qubit idea
- \* Current biased JJ, open or in an rf-squid design



- \* These local minima can host metastable states which, if long lived, can be used as qubits
- \* When a state decays into the continuum  $\dot{\phi} = C I$  leading to a growing  $V \propto t$  voltage and a large release of energy
- \* Changing  $I$  modifies the decay rates, so that one can distinguish  $|1\rangle$  from  $|0\rangle$

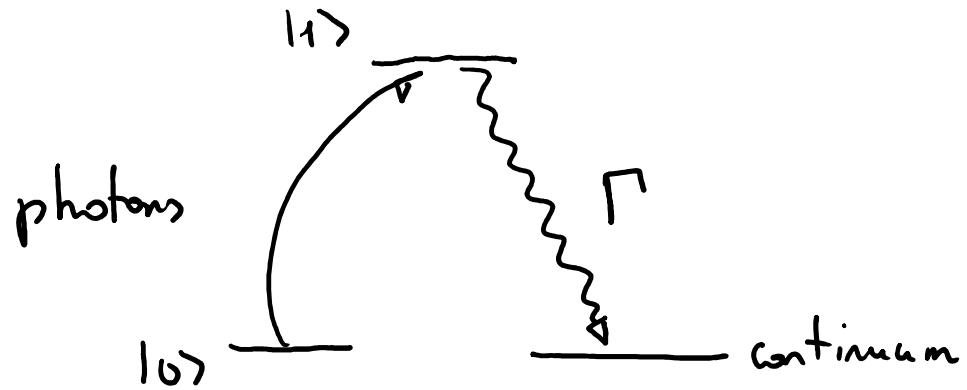
$$\mathcal{L} \sim \frac{1}{2C} (\dot{\phi})^2 - E_{\text{ind}}$$

$$E_{\text{ind}} \sim -E_J \cos\left(\phi \frac{2n}{\Phi_0}\right) - I \phi$$

# Phase qubit applications

jueves, 25 de febrero de 2016 11:29

Quantum opticians viewpoint



The phase qubit can be excited by incoming photons and quickly relax to the continuum, releasing measurable energy /voltage.

- ↳ photo detector
- ↳ photon counter

# Qubit comparison

jueves, 25 de febrero de 2016 11:16

## transmon

### pros

- reproducibility
- simple design & fab
- macroscopic if needed
- now switchable "g"

### cons

- weakly anharmonic
- only  $g \ll$  anharmonicity  $\sim 5\% \omega_0$

## flux qubit

- huge anharmonicity
- no bounds on coupling
- switchable coupling

- exponential dependence of  $D$  on  $E_J$ 
  - ↳ difficult fabrication & reproducibility
  - ↳ noise in  $E_J$

## phase qubit

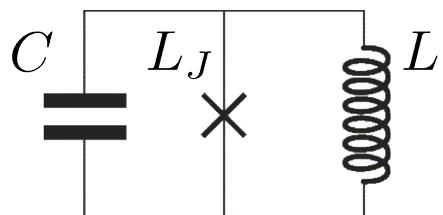
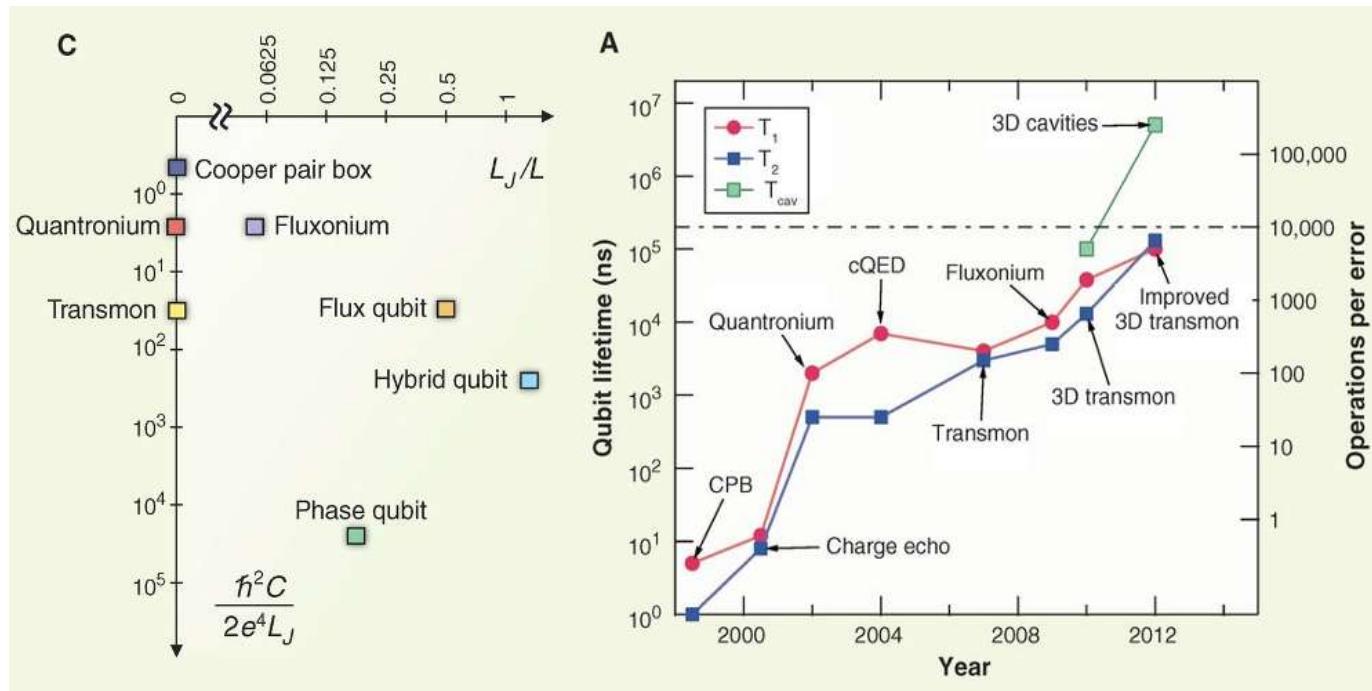
- metastable states

- metastable states
- weakly anharmonic

# Technology progress

jueves, 25 de febrero de 2016 11:32

*Superconducting Circuits for Quantum Information: An Outlook*  
M. H. Devoret, R. J. Schoelkopf, Science 339, 1169-1174 (2013)

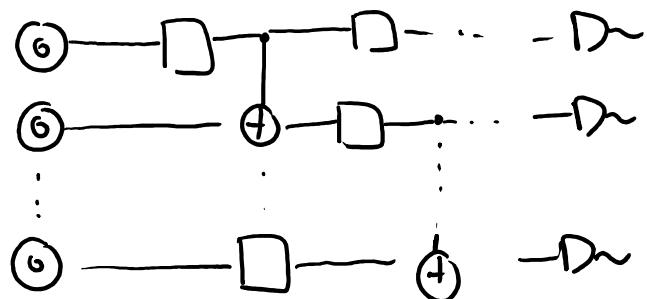


There exist many ingenious designs for qubits with these simple elements, but existing ones might be enough

# Quantum computing models

domingo, 28 de febrero de 2016 17:13

## a) Gate circuits



## c) Adiabatic quantum computing

- 1) Encode solution of a problem as a ground state of an energy function  $H_p |4_p\rangle = E_p |4_p\rangle$
- 2) Find a simple Hamiltonian  $H_0$  whose g. state  $|4_0\rangle$  is easy to prepare
- 3) Implement experimentally  $|4_0\rangle$  and  $H(t) = (1 - t/T) H_0 + t/T H_p \quad (t \in [0, T])$
- 4) If  $\min_t (\text{gap}(H(t))) \gg O\left(\frac{\epsilon}{T}\right)$ ,  $|4(t)\rangle$  will evolve from  $|4(0)\rangle = |4_0\rangle$  to  $|4(t)\rangle \approx |4_p\rangle$   
(... and TQI, and other models)

## b) Measurement based

- 1) Create entangled state resource
- 2) Perform measurements and feedback on a subset of qubits
- 3) Get outcome by measuring rest of qubits

[All have same comp. power !!!]

# Model problems

domingo, 28 de febrero de 2016 17:21

- All quantum computations can be recasted in AQC form, using a physical system as "clock" whose state counts the # gates realized
- But there are problems that have simpler interactions making  $H_p$  experimentally "simple".

a) Spin glasses  $H_p = \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z$   $J \in [-1, 1]$ , long range

↳ approximate models for protein folding

b) SAT problems  $H_p = \sum_{\text{sentences}} \underbrace{\prod^{(n)} (s_{i_1}, s_{j_1}, s_{k_1})}_{f \in \{0,1\} \text{ outcome of a logical sentence}}$   $s \in \{0,1\}$

↳ 3-SAT and higher are proven NP-complete = cheap to verify the solution

and all other NP problems map to them

↳ Unless  $NP = P$ , we expect exp. large resources to solve them deterministically

# Typical scenario and difficulties

domingo, 28 de febrero de 2016 17:35

- 1) We normally take  $H_0 = -\sum \sigma_i^x$ ; because its ground state contains all possible configurations  $|\Psi_0\rangle = \bigotimes_{i=1}^n \frac{1}{2} (|0\rangle + |1\rangle)$

- 2) The actual problem is that  $H_0 \lambda + (1-\lambda) H_P$  <sup>(\*)</sup> may experience a phase transition for  $\lambda \in [0,1]$

$$\min |\text{gap}(H(\lambda))| \sim \text{exponential}(-\text{prob. size}) !!!$$

This implies slow time evolution  $\gg$  coherence times of experiment  $\Rightarrow$  dephasing

- 3) Moreover,  $H_P$  demands a high connectivity to build interesting (= hard) problems

(\*) Other schedules  $A(t) H_0 + B(t) H_P$  are possible, provided  $B(0)=0$ ,  $A(T)=0$   
but they can be slightly more complicated to analyze

# Annealing

domingo, 28 de febrero de 2016

19:43

Simulated Annealing is an stochastic classical optimization algorithm which simulates the trajectories of a system under a progressively lower temperature

Quantum Annealing is similar to S.A. but uses a model w. quantum fluctuations such as  $H = \sum J \sigma_i^z \sigma_j^z + \sum h_i \sigma_i^x$ , where " $\sigma^x$ " allows tunneling b/w. configurations

Adiabatic Computation follows none of these models and works only w. pure ground states

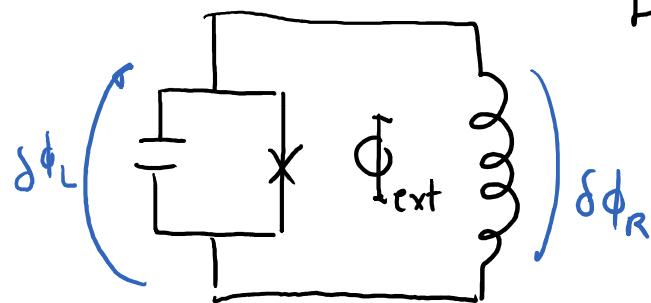
Real experiments depart from AQC and approximate either QA or SA.

↳ If  $T$  is very high, we expect the system to be incoherent  $\Rightarrow$  SA

↳ If  $T$  is moderate, we might witness some coherence & Q. tunneling  $\Rightarrow$  QA

# rf-squid

lunes, 22 de febrero de 2016 9:18

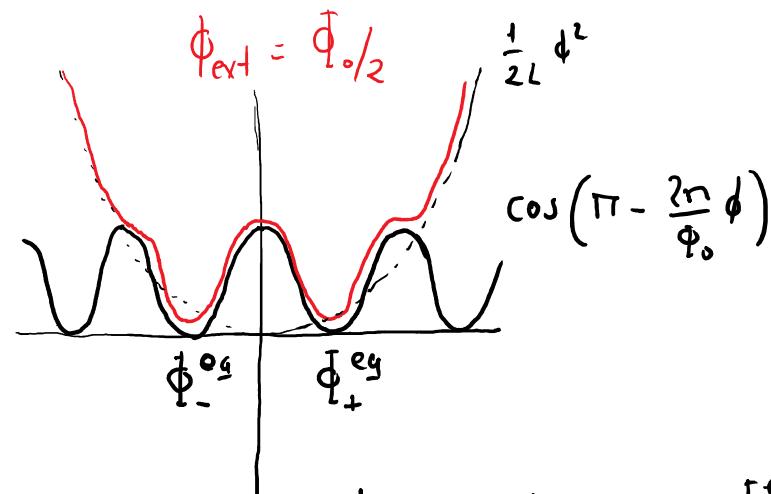
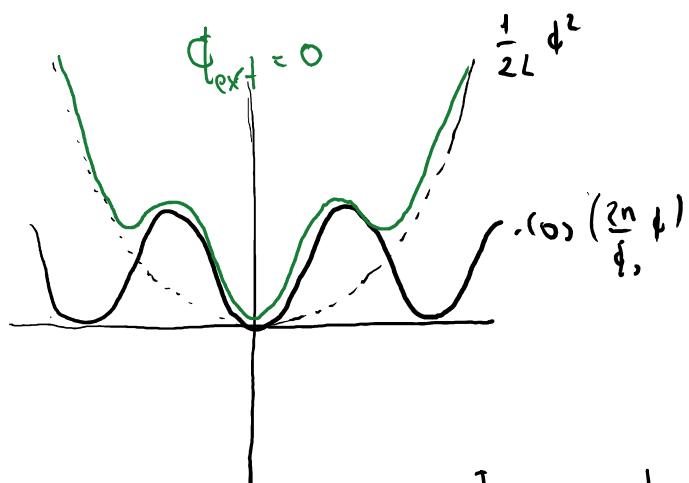


$$\delta\phi_L + \delta\phi_R = \Phi_{ext} + 2\pi n \frac{\phi}{\phi_0}$$

Hamiltonian  $\mathcal{H} \sim \frac{1}{2C} q^2 + \frac{1}{2L} (\phi)^2 - E_J \cos \left[ \frac{2\pi}{\phi_0} (\Phi_{ext} - \phi) \right]$

When  $\Phi_{ext} = 0$ , the minimum energy is obtained for  $\phi \approx 0$ , no flux difference

$$I(\Phi_{ext}=0) = I_c \sin \left( \frac{2n}{\phi_0} (\phi=0) \right) = 0$$

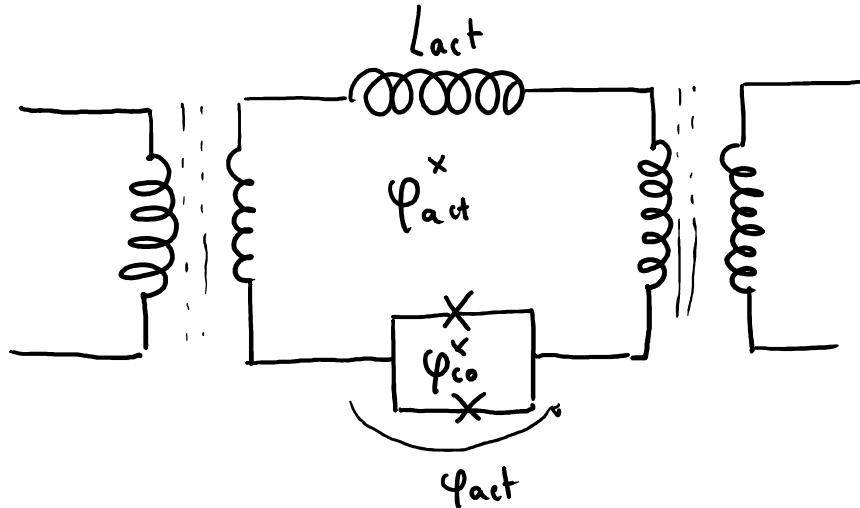


But when  $\Phi_{ext} \sim \frac{\phi_0}{2}$  we have two quasi-degenerate ground states  $\phi_{\pm}^{eq}$

# rf-squid as tuneable coupler

domingo, 28 de febrero de 2016

15:35



We assume that the dc-squid part has a very tight inductive potential ( $L_c \ll L_{act}$ )

In this case we only have one effective phase variable,  $\varphi_{act}$ , the average over both branches

$$H_n = \frac{q_{act}^2}{2C} + \frac{1}{2L_{act}} \left( \frac{\Phi_0}{2\pi} \right)^2 (\varphi_{act} - \varphi_{act}^x)^2 - 2E_J \cos(\varphi_{co}/2) \cos(\varphi_{act})$$

We can improve the theory considering asymmetries in dc-squid

$$E_{ind} = U_{act} \left\{ \frac{(\varphi_{act} - \varphi_{act}^x)^2}{2} - \beta_{eff} \cos(\varphi_{act} - \varphi^*) \right\} \quad \leftarrow \varphi^* = -\arctan \left[ \frac{I_{c-}}{I_{c+}} \tan \frac{\varphi_{co}^x}{2} \right]$$

$I_{c\pm} = I_{c1} \pm i I_{c2}$

$\beta_{eff} = \frac{2\pi}{\Phi_0} L_{act} I_{c+} \cos \left( \frac{\varphi_{co}^x}{2} \right) \sqrt{1 + \left[ \frac{I_{c-}}{I_{c+}} \tan \left( \frac{\varphi_{co}^x}{2} \right) \right]^2}$

(asymmetry of dc-squid)

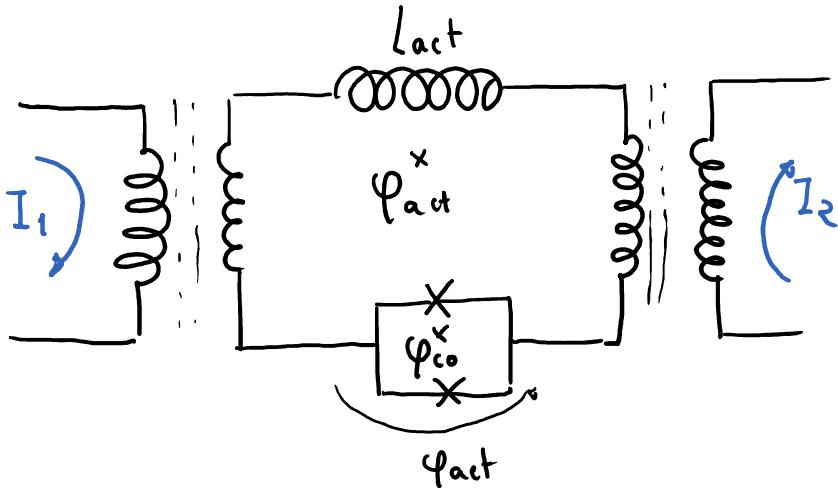
# Mutual inductance

domingo, 28 de febrero de 2016 15:59

The basic premise is that

1) Each external element close to the coupler contributes to  $\varphi_{act}^x$

2) The coupler plasma freq. is so large that it will rapidly follow those changes



$$H = H_1(z_1) + H_2(z_2) + H_{rf}(z_1, z_2) \xrightarrow{\text{adiab}} H_1(z_1) + H_2(z_2) + E_{GS}^{rf}(z_1, z_2)$$

3) We replace in the Hamiltonian the eff. energy of the rf-squid under those external conditions.

$$E_{rf}^{GS}(z_1, z_2) \sim E_{rf}^{GS}(\varphi_{act}^{eq.}) + \frac{\partial E_{rf}^{GS}}{\partial \varphi_{act}}(\varphi_{act}^{eq.}) \cdot \underbrace{\frac{\partial \varphi_{act}^{eq}}{\partial \varphi_{act}^x} \cdot \left[ \frac{M_1 z_1 + M_2 z_2}{\Phi_0/2n} \right]}_{\delta \varphi_{act}^x} + \frac{1}{2} \frac{\partial^2 E_{rf}^{GS}}{\partial \varphi_{act}^2}(\varphi_{act}^{eq.}) \left[ \delta \varphi_{act}^x \right]^2$$

zero by definition of GS

# Mutual inductance (2)

domingo, 28 de febrero de 2016 16:30

$$E(\varphi_{act}^x) = \frac{1}{L_{act}} \left( \frac{\Phi_0}{2\pi} \right)^2 \left[ \left( \frac{\varphi_{act}^{eq} - \varphi_{act}^x}{2} \right)^2 - \beta_{eff} (\varphi_{co}^x) \cos (\varphi_{act}^{eq} - \varphi_{act}^x) \right]$$

where  $\varphi_{act}^{eq}$  depends on  $\varphi_{act}^x$  and is given by

$$\left\{ \begin{array}{l} \frac{\partial E}{\partial \varphi_{act}} = 0 \Rightarrow \varphi_{act}^{eq} - \varphi_{act}^x + \beta_{eff} \sin (\varphi_{act}^{eq} - \varphi_{act}^x) = 0 \\ \frac{\partial \varphi_{act}}{\partial \varphi_{act}^x} = [1 + \beta_{eff} \cos (\varphi_{act}^{eq} - \varphi_{act}^x)]^{-1} \end{array} \right.$$

$$E(\varphi_{act}^x) \sim \frac{1}{L_{act}} \left( \frac{\Phi_0}{2\pi} \right)^2 \left\{ \frac{1}{2} \beta_{eff}^2 \sin (\varphi_{act}^{eq} - \varphi_{act}^x) - \beta_{eff} \cos (\varphi_{act}^{eq} - \varphi_{act}^x) \right\}$$

$$E \sim E(0) + \frac{\partial E}{\partial \varphi_{act}^{eq}} \cdot \frac{\partial \varphi_{act}^{eq}}{\partial \varphi_{act}^x} \cdot \delta \varphi + \frac{1}{2} \delta \varphi^2 \left[ \frac{\partial E}{\partial \varphi_{act}^{eq}} \cdot \frac{\partial^2 \varphi_{act}^{eq}}{\partial \varphi_{act}^{eq}^2} + \frac{\partial^2 E}{\partial \varphi_{act}^{eq}^2} \left( \frac{\partial \varphi_{act}^{eq}}{\partial \varphi_{act}^x} \right)^2 \right]$$

$$E \sim \frac{1}{L_{act}} \left( \frac{\Phi_0}{2\pi} \right)^2 \cdot \left[ \frac{1}{2} \delta \varphi^2 \frac{\beta \cos (\varphi_{act}^{eq} - \varphi_{act}^x)}{\beta \cos (\varphi_{act}^{eq} - \varphi_{act}^x) + 1} + \beta \delta \varphi \sin (\varphi_{act}^{eq} - \varphi_{act}^x) \right]$$

# Mutual inductance (3)

Lunes, 29 de febrero de 2016 8:42

An alternative, qualitative approach.

1. A current  $I_p$  in the top causes a flux  $M_2 I_p$  on the 2nd device

2. The current  $I_p$  depends on the first device as

$$\frac{\partial I_p}{\partial \phi_{act}^x} \times \underbrace{M_1 I_L}_{\text{flux induced}}$$

3. Mutual inductance thus  $M_1 M_2 \frac{\partial I_p}{\partial \phi_{act}^x}$

4.  $I_p = \frac{\Phi_0}{2\pi} \frac{1}{L_{act}} \cdot \beta_{eff} \sin(\varphi_{act}^{eq} - \varphi_{act}^o)$

5.  $\varphi_{act}^{eq} - \varphi_{act}^x + \beta_{eff} \sin(\varphi_{act}^{eq} - \varphi_{act}^o) = 0 \Rightarrow$

$$\frac{\partial \varphi_{act}^{eq}}{\partial \phi_{ext}^x} = \frac{1}{1 + \rho \cos(\varphi_{act}^{eq} - \varphi_{act}^o)} \frac{2\pi}{\Phi_0}$$

6.  $M_1 M_2 \cdot \frac{1}{L_{act}} \cdot \frac{\rho \cos(\varphi_{act}^{eq} - \varphi_{act}^o)}{1 + \rho \cos(\varphi_{act}^{eq} - \varphi_{act}^o)}$

# Experimental calibration

domingo, 28 de febrero de 2016 16:04

- The phase variations are

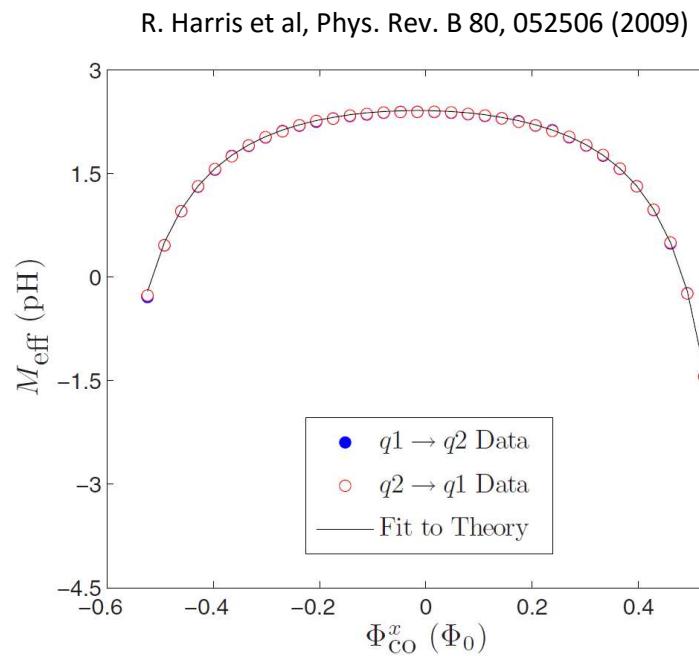
$$\delta\varphi^x = \left( M_1 I_1 + M_2 I_2 \right) \frac{2\pi}{\Phi_0}$$

- Mutual inductance terms

$$E \sim M_{\text{eff}} \cdot I_1 I_2$$

$$M_{\text{eff}} = \frac{\Phi_0}{2\pi L_{\text{act}}} \cdot M_1 M_2 \cdot \frac{\beta_{\text{eff}}(\varphi_{\text{co}}^x)(0)(\varphi^{\text{eq}} - \varphi^0)}{\beta_{\text{eff}}(\varphi_{\text{co}}^x)(0)(\varphi^{\text{eq}} - \varphi^0) + 1}$$

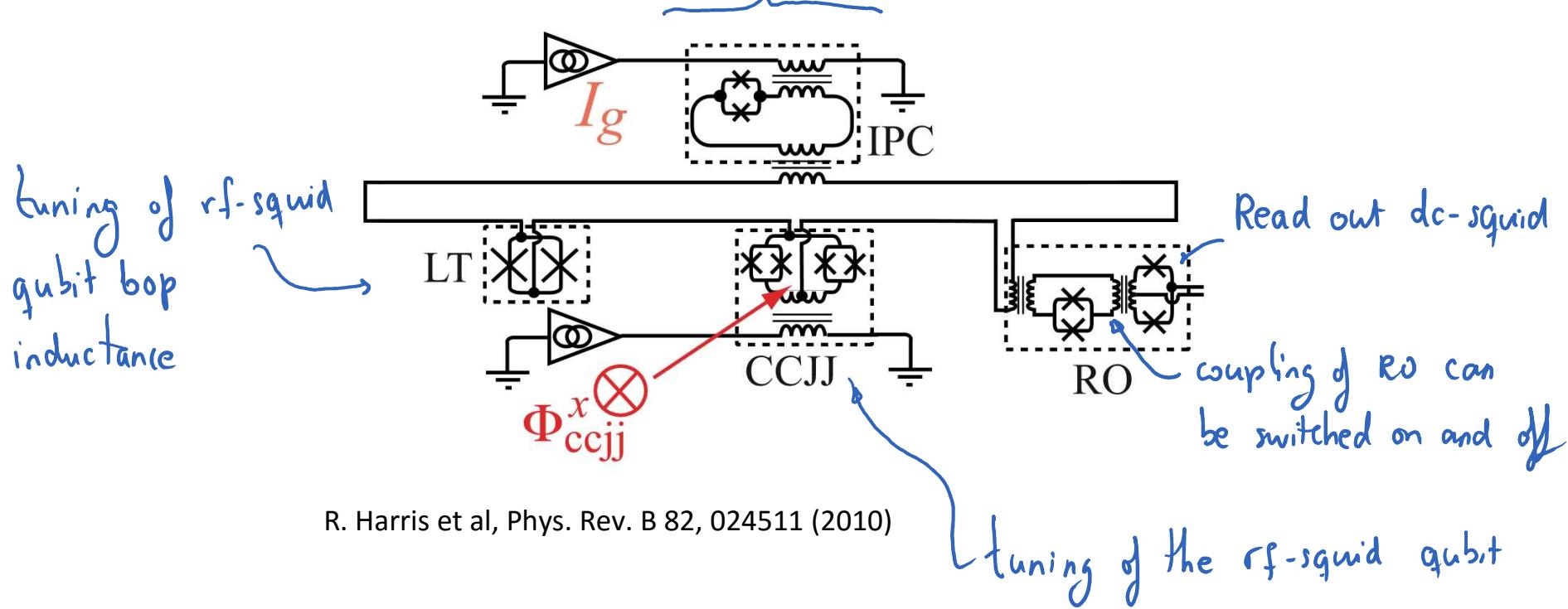
- Now  $\beta_{\text{eff}}(\varphi_{\text{co}}^x)$  is tuneable in magnitude and sign, but it is constrained to avoid bistability :  $-\min[1, \beta_{\text{eff}}(0)] \leq \beta_{\text{eff}}(\varphi_{\text{co}}^x) \leq \beta_{\text{eff}}(0)$



# 1 qubit control

domingo, 28 de febrero de 2016 9:36

Coupling circuit to current or  
flux source: used also for qb-qb coupling



# "Magnetic memory" approach

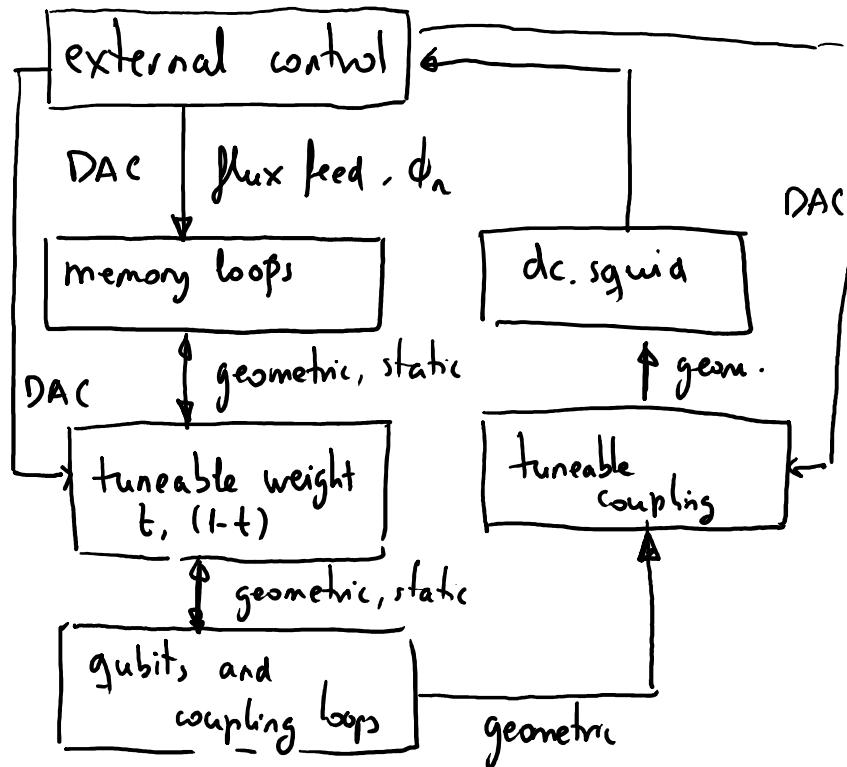
domingo, 28 de febrero de 2016 15:20

- \* The system needs a lot of control fluxes : qubit parameters, couplers, etc
- \* If designing adiabatic computers, the parameters can be fixed and only modulated

$$H(t) = H_0 \cdot t + (1-t) H_1,$$

two sets of fluxes

- \* This is achieved by storing the fluxes in "memory" coils and afterwards coupling those coils to the actual qubits / couplers



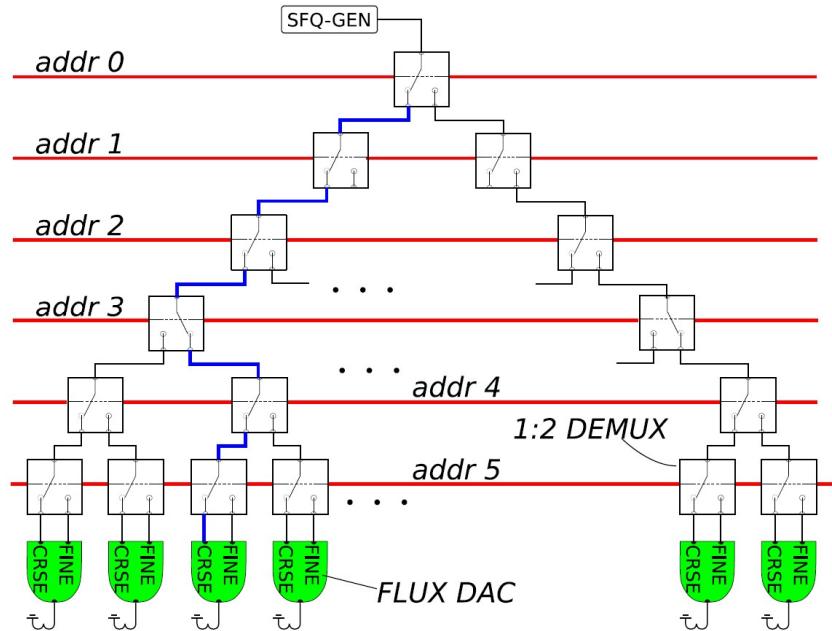
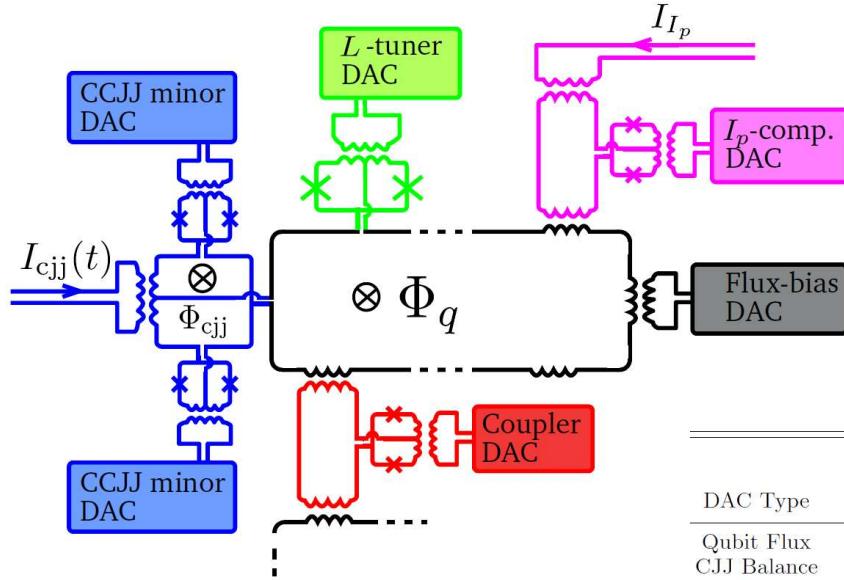
# DAC (digital to analog converter)

domingo, 28 de febrero de 2016 12:00

TABLE I. Parts count vs. number of unit cells.

Unit Cells	Qubits	Couplers	DACS	JJs
1	8	16	56	1500
4	32	72	232	6000
16	128	328	968	24000
64	512	1416	3976	96000
256	2048	5896	16136	384000

M.W. Johnson et al  
Supercond. Sci. Technol. 23 (2010) 065004



- 1) Need for many tuneable control lines
- 2) Need for fine tuning and t-dep control

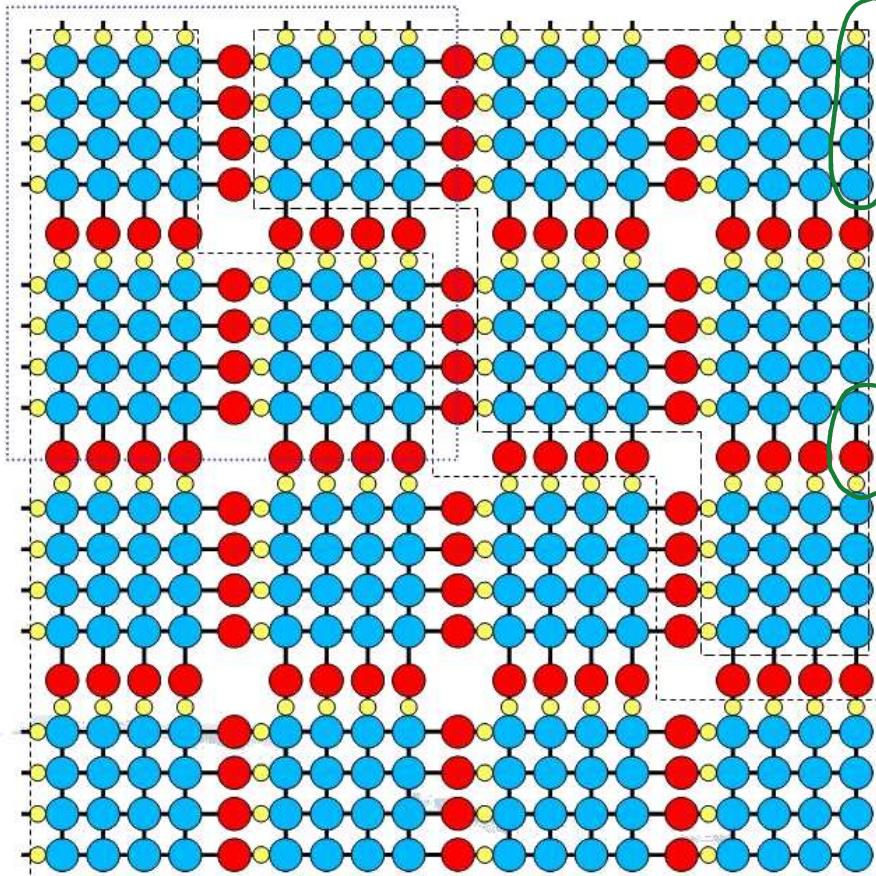
TABLE II. Designed flux ranges and minimal flux steps by DAC type

DAC Type	Span	min $\Delta\Phi$	Max # $\Phi_0$		COARSE/FINE Ratio
			COARSE	FINE	
Qubit Flux	25.5 m $\Phi_0$	0.1 m $\Phi_0$	17	17	14.1
CJJ Balance	66.1 m $\Phi_0$	0.4 m $\Phi_0$	17	17	14.1
L-Tuner	0.465 $\Phi_0$	1.1 m $\Phi_0$	40	10	10.7
Coupler	0.968 $\Phi_0$	2.2 m $\Phi_0$	40	10	10.6

# Problem topology

domingo, 28 de febrero de 2016 12:23

128 qubit C<sub>4</sub> Chimera graph



(c) D-Wave

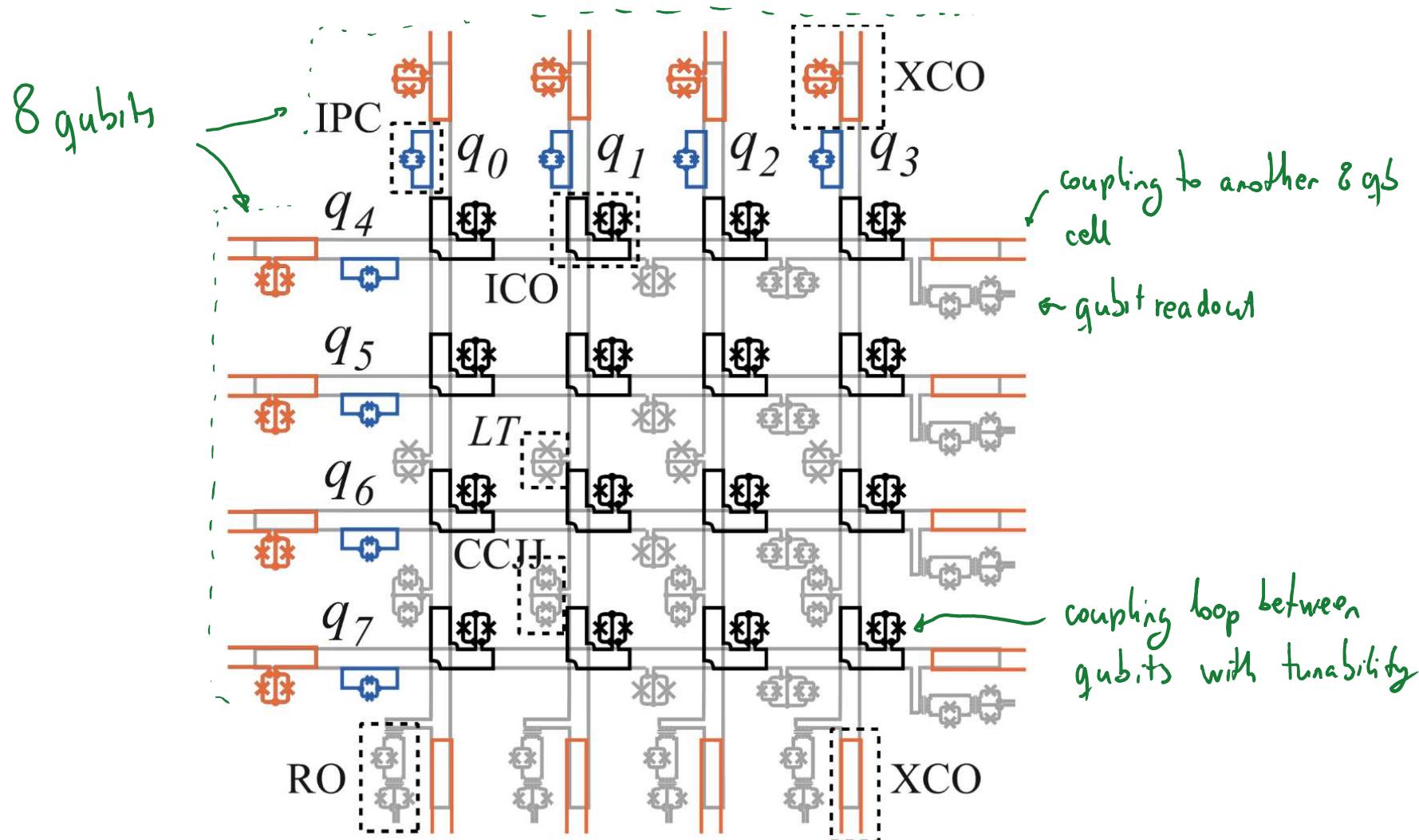
Ideally we would like any-to-any connectivity to implement arbitrary complex problems

This would require either

- Arbitrarily long couplers connecting all qubits
- An embedding or encoding that maps a problem w. long-range coupling to a lattice w. lesser connectivity

# 8 qubit planar arrangement

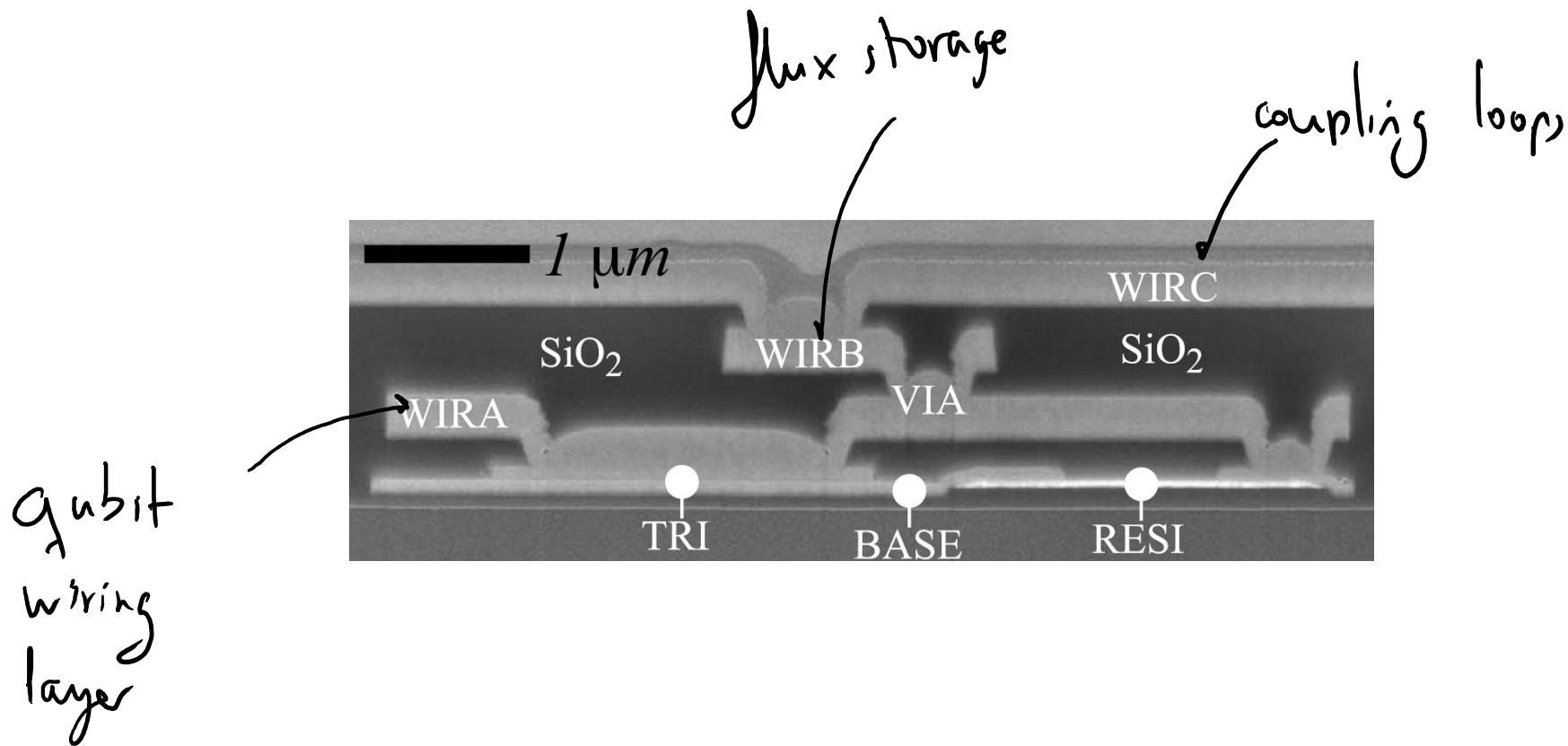
domingo, 28 de febrero de 2016 9:34



R. Harris et al, Phys. Rev. B 82, 024511 (2010)

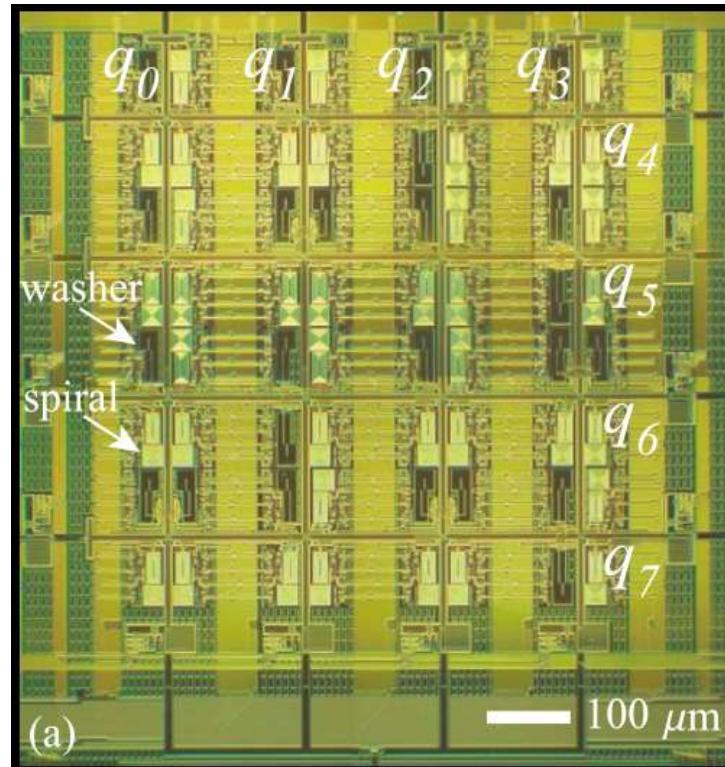
# 3D architecture

domingo, 28 de febrero de 2016 9:45

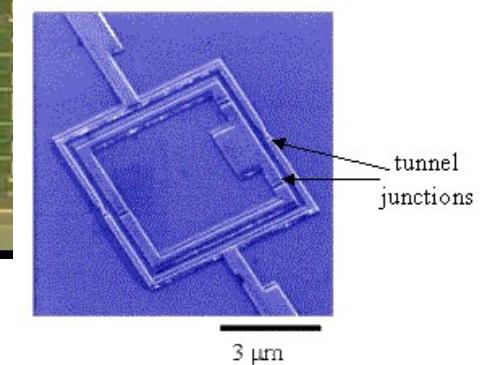
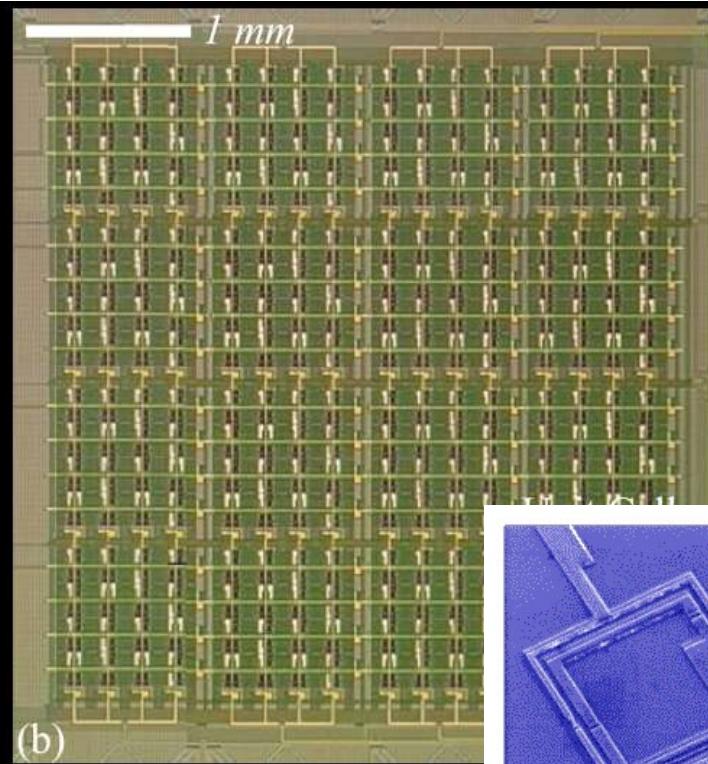


# Final chip (1 cell)

domingo, 28 de febrero de 2016 11:52



R. Harris et al, Phys. Rev. B 82, 024511 (2010)



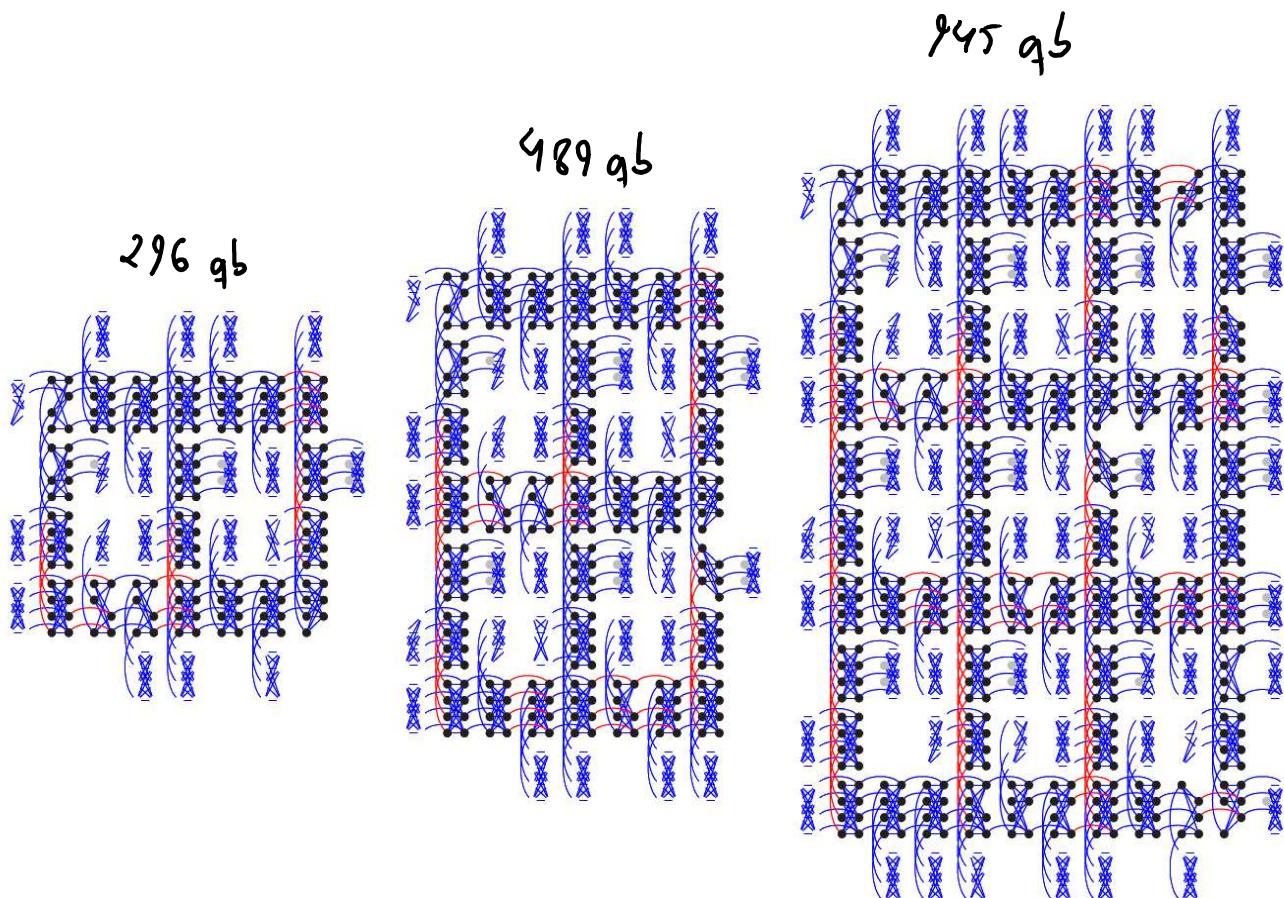
C van der Waal et al Science 290, 773 (2000)

# Actual problem topology

domingo, 28 de febrero de 2016 19:56

Having a given topology  
does not force us to  
use all of it

Also, in a real setup, not  
all qubits have the same  
quality and 10% might  
have to be ignored

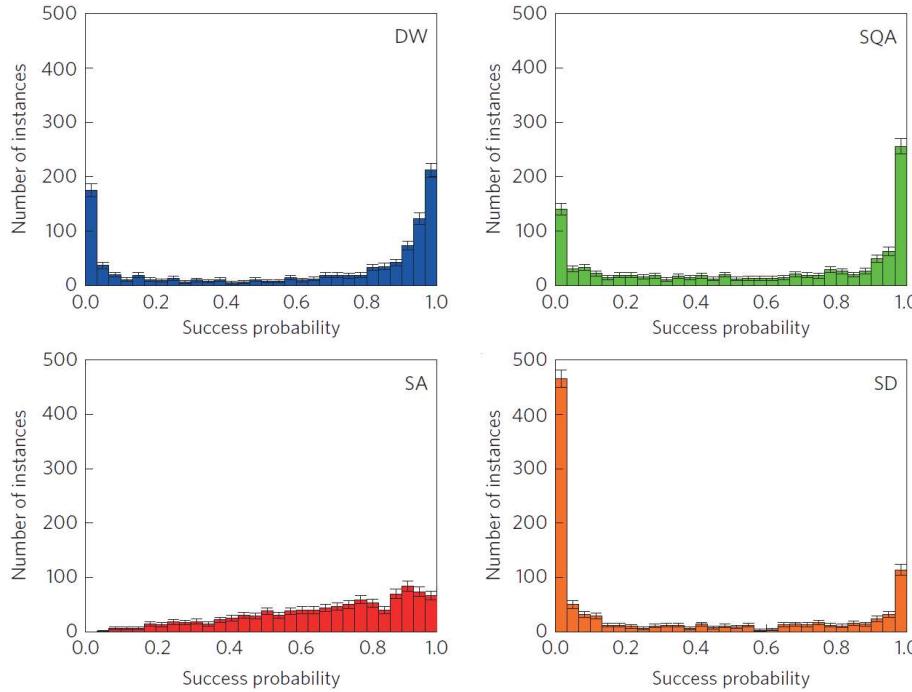


V. S. Denchev arXiv:1512.02206v4

D-Wave 2x design

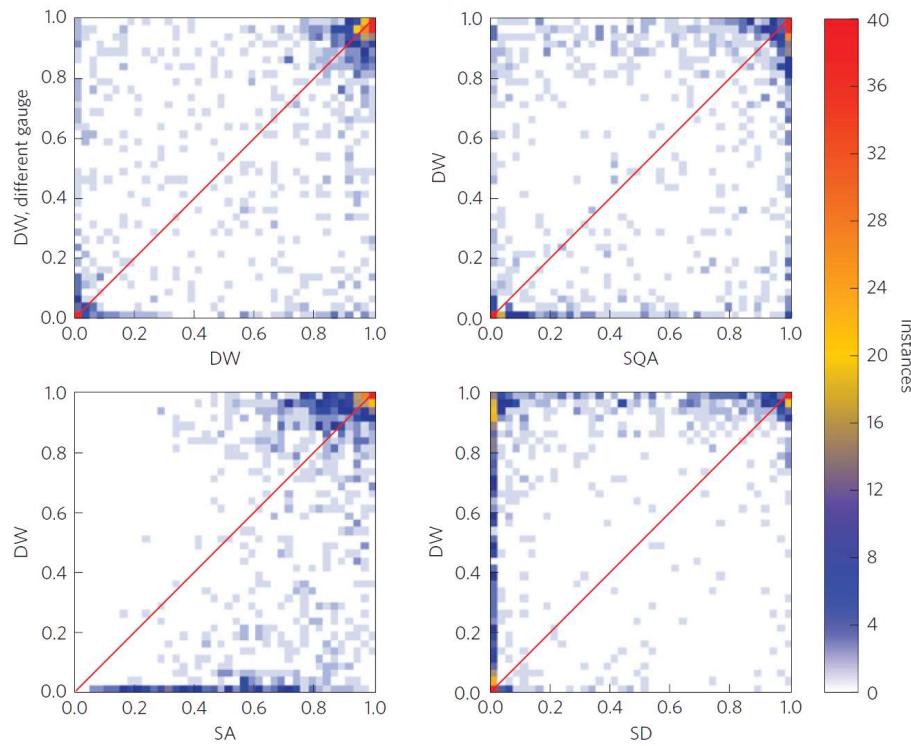
# Comparison with classical annealing

domingo, 28 de febrero de 2016 19:06



S. Boixo et al, Nat. Phys. 10, 218–224 (2014)

The statistics of success in various instances is consistent with simulated quantum annealing

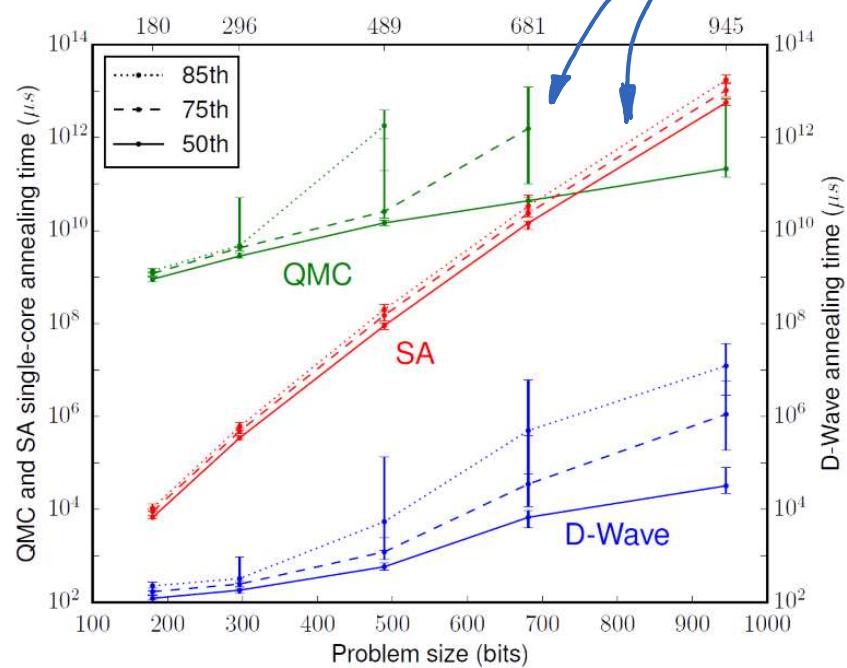


# Custom design problems

domingo, 28 de febrero de 2016 18:38

Quantum Annealing is expected to provide an advantage for problems that involve tunneling bw. nearby solutions but with a potential barrier too high for thermal activation

with optimal running times & Temp



V. S. Denchev arXiv:1512.02206v4

$$H_P = H_P^1 + H_P^2 + H_P^{1,2}$$

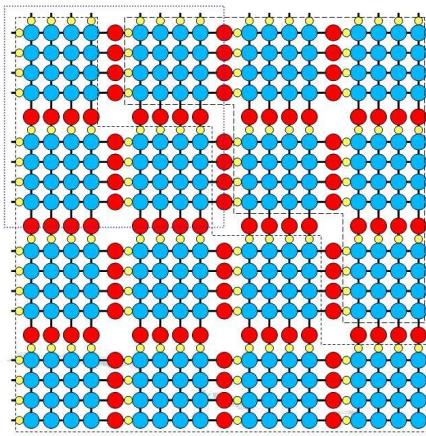
$$H_P^k = -J \sum_{\langle j,j' \rangle \in \text{intra}} \sigma_{k,j}^z \sigma_{k,j'}^z - \sum_{j=1}^8 h_k \sigma_{k,j}^z$$

$$H_P^{1,2} = -J \sum_{j \in \text{inter}} \sigma_{1,j}^z \sigma_{2,j}^z .$$

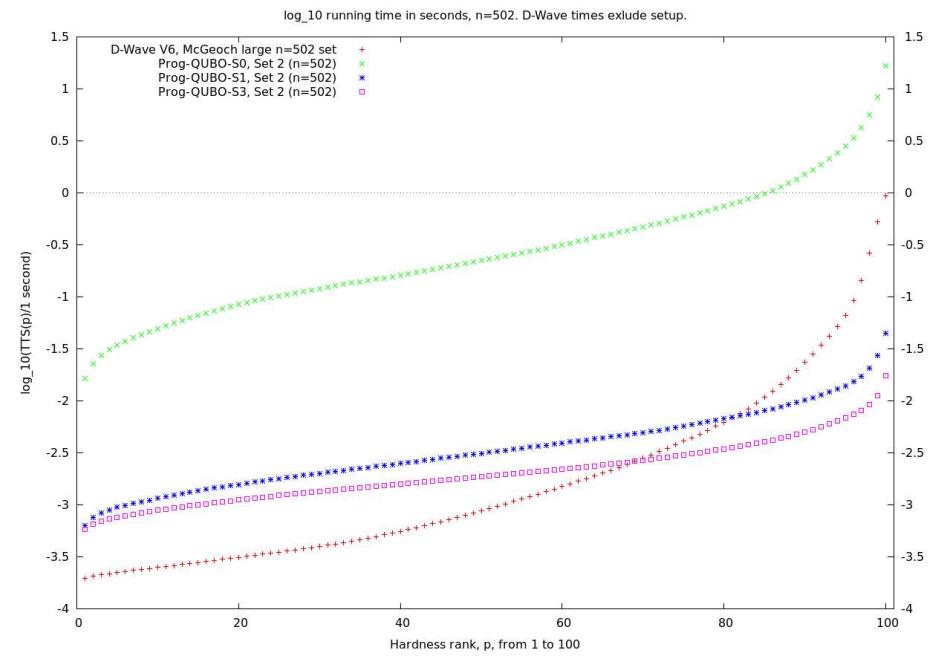
# Ad-hoc algorithms

domingo, 28 de febrero de 2016 20:07

128 qubit C<sub>4</sub> Chimera graph



Because of the cluster topology of the chimera graph, one can design algorithms that replace each cluster with two larger spins ( $d = 2^4 = 16$ ) and use those classical variables for S. Annealing



This is what A. Selby did, producing a simple code that solves many of the D-Wave problems efficiently