A carefully studied circuit

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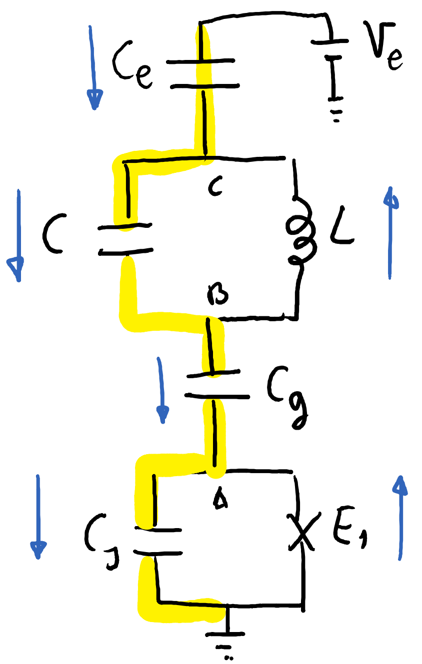

Node A: $C_J \ddot{\phi}_A - C_g(\ddot{\phi}_B - \ddot{\phi}_A) = I_c \sin(-\phi_A)$

Node B: $C(\ddot{\phi}_c - \ddot{\phi}_B) - C_g(\ddot{\phi}_B - \ddot{\phi}_A) = \frac{1}{L}(\phi_B - \phi_c)$

Node C: $C(\ddot{\phi}_c - \ddot{\phi}_B) - C_e(\dot{V}_e - \ddot{\phi}_c) = \frac{1}{L}(\phi_B - \phi_c)$

$$\mathcal{L} = \frac{1}{2} C_J (\dot{\phi}_A)^2 + \frac{1}{2} C_g (\dot{\phi}_A - \dot{\phi}_B)^2 + \frac{1}{2} C (\dot{\phi}_B - \dot{\phi}_c)$$
$$+ \frac{1}{2} C_e (\dot{\phi}_c - V_e)^2 - \frac{1}{2L} (\phi_c - \phi_B)^2 - E_J \cos(\phi_A \, 2\pi/\phi_0)$$

Change of variables: $\dot{\phi}_c = \dot{\tilde{\phi}}_c + V_e \qquad \phi_B = \tilde{\Phi}_c + V_e + \phi_{LC}$

$$\mathcal{L} = \frac{1}{2} C_J \dot{\phi}_A^2 + \frac{1}{2} C_g (\dot{\phi}_A - \dot{\phi}_{LC} - \dot{V}_e - \tilde{\Phi}_c)^2 + \frac{1}{2} \dot{\phi}_{LC}^2 C + \frac{1}{2} C_e \dot{\tilde{\Phi}}_c^2 - \frac{1}{2L} \phi_{LC} + E_J \cos\left(\frac{\phi_A 2\pi}{\phi_0}\right)$$

We can see a qubit lagrangian, a resonator, a variable that has no dynamics $\tilde{\phi}_c$ and a qubit-resonator coupling.

Total Hamiltonian

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We start by eliminating $\tilde{\phi}_c = 0$ (or any other constant)

$$\left.\begin{aligned} q &= \frac{\delta \mathcal{L}}{\delta \dot{\phi}_A} = (C_J + C_g)\,\dot{\phi}_A - C_g(\dot{\phi}_{LC} + V_e) \\ Q_{LC} &= \frac{\delta \mathcal{L}}{\delta \dot{\phi}_{LC}} = (C + C_g)\,\dot{\phi}_{LC} + C_g(V_e + \dot{\phi}_a) \end{aligned}\right\} \text{invert \& compute } q\dot{\phi}_{LC} + Q\dot{\phi}_A - \mathcal{L}$$

After a tedious calculation

$$C_\Pi^2 = C_g C_J + C_J C + C C_g$$

$$H = \frac{1}{2}\begin{pmatrix} q & Q_{LC}\end{pmatrix}\begin{pmatrix} \frac{C + C_g}{C_\Pi^2} & \frac{C_g}{C_\Pi^2} \\ \frac{C_g}{C_\Pi^2} & \frac{C_g + C_J}{C_\Pi^2} \end{pmatrix}\begin{pmatrix} q \\ Q_{LC}\end{pmatrix} + \frac{C C_g}{C_\Pi^2} V_e q - \frac{C_g C_J}{C_\Pi^2} V_e Q_{LC}$$

$$+ \frac{1}{2L}\phi_{LC}^2 - E_J \cos\left(\frac{\dot{\phi}_A}{\phi_0} 2\pi\right)$$

Interpretation & linear coupling

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a) Linear coupling between qubit and cavity $q\,Q_{LC}\quad \frac{C_g}{C_\Pi^2}$

b) Renormalization of qubit and cavity capacitances

c) Both objects influenced by driving V_e

d) If V_e is constant, it does not affect the cavity $\Rightarrow$ it can be used to reach qubit degeneracy

e) We can take some limits, such as large "C" to neglect the influence of these terms:

$$H \xrightarrow[C\to\infty]{} \frac{1}{2}(q\ \ Q)\begin{pmatrix} \frac{1}{C_\Sigma} & \frac{C_g}{C_\Sigma C} \\ \frac{C_g}{C_\Sigma C} & \frac{1}{C} \end{pmatrix}\begin{pmatrix} q \\ Q \end{pmatrix} + \frac{C_g}{C_\Sigma} V_e \cdot q + \dots \qquad C_\Sigma = C_g + C_J$$

$$\sim H_{qb} + H_{LC} + \frac{C_g}{C_\Sigma}\, q \left[\frac{1}{C} Q + V_e \right]$$

$$\sim H_{qb} + H_{2c} + \frac{C_g}{G_z} g \left[\frac{1}{c} \dot{Q} + v e \right]$$

A2 terms and ultrastrong coupling

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Light-matter decoupling and A2 term detection in superconducting circuits
J. J. García-Ripoll, B. Peropadre & S. De Liberato Scientific Reports 5, 16055 (2015)

If we consider the cavity renormalization

$$H \sim \frac{1}{2C_\Sigma}\left[q_J - C_g \partial_t \phi(0,t)\right]^2 - E_J \cos(2e\phi_J/\hbar)$$

$$+ \int_{-L/2}^{L/2}\left[\frac{p(x,t)^2}{2C_0} + (\partial_x \phi(x,t))^2\right] dx$$

This implies that we can write

$$H = H_{qb} + H_{coupling} + H'_{LC}$$

$$H'_{LC} = \int_{-L/2}^{L/2}\left[\frac{1}{2C_0}p(x,t)^2 + \underbrace{\frac{C_g^2}{2C_\Sigma}\partial_t \phi(0,t)^2}_{} + (\partial_x \phi(x,t))^2\right] dx$$

This term tends to expel the field from around the qubit ⇒ coupling is suppressed

Idea for experiments

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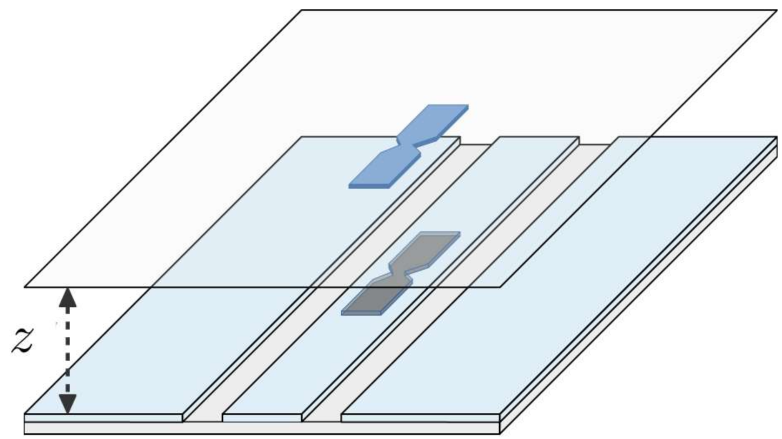

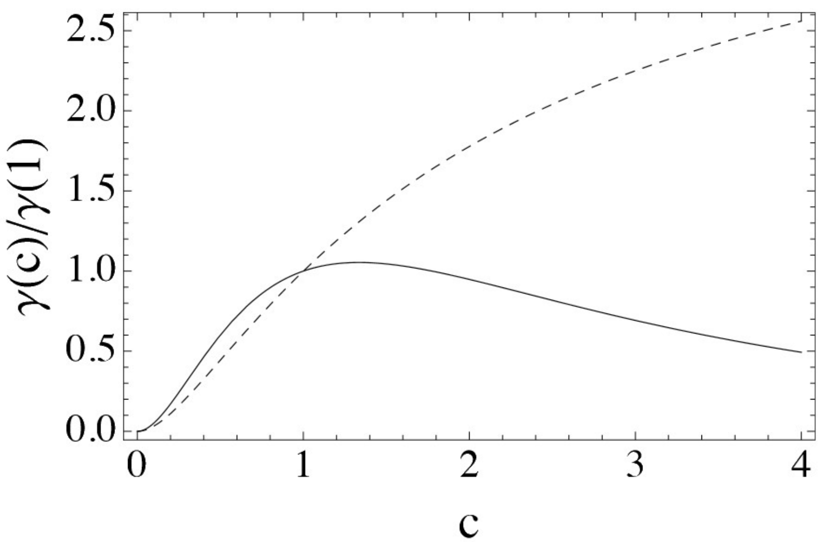

Jaynes-Cummings

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$$H \sim H_0 + g(\sigma^+ + \sigma^-)(a + a^+)$$

We apply the rotating wave approx.

$$H = \frac{\Delta}{2}\sigma^z + g(\sigma^+ a + \sigma^- a^+) + \omega a^+ a$$

The Hamiltonian becomes box. diagonal $|g/e\rangle \otimes |n\rangle$

$$\begin{pmatrix} -\Delta/2\,|g,0\rangle\langle g,0| & 0 & & 0 \\ 0 & \left(-\frac{\Delta}{2}+\omega\right)|g1\rangle\langle g1| & g\,|g0\rangle\langle e0| & 0 \\ & g\,|e0\rangle\langle g1| & \frac{\Delta}{2}\,|e0\rangle\langle e0| & \\ 0 & 0 & & \ddots \end{pmatrix}$$

$|e\, n-1\rangle \quad \frac{\Delta}{2} + \omega(n-1)$

$g\sqrt{n} \qquad \updownarrow \delta \qquad \delta = (\omega - \Delta)$

$|g\, n\rangle \quad -\frac{\Delta}{2} + \omega n$

Rabi oscillations

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$|e\, n-1\rangle \quad \frac{\Delta}{2} + \omega(n-1)$

$g\sqrt{n} \qquad \delta$

$|g\, n\rangle \quad -\frac{\Delta}{2} + \omega n$

$\delta = (\omega - \Delta) \qquad H_n \sim \frac{\delta}{2}\sigma^z + \sqrt{n}\, g\, \sigma^x \qquad (\hbar = 1)$

Integrate Schröedinger eq.: $U(t) = e^{-i(\vec{n}\cdot\vec{\sigma})\cdot\theta}$ $\qquad \theta = t\sqrt{\frac{\delta^2}{4} + n g^2} \approx t\tilde{\Delta}$

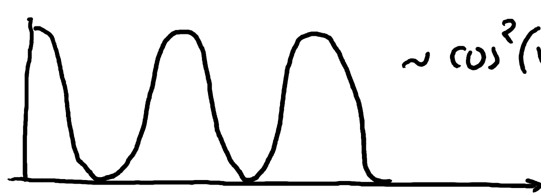
Trick: $e^{i(\vec{n}\cdot\vec{\sigma})\theta} = \sum_n \frac{(i\theta)^n}{n!}\cdot(\vec{n}\cdot\vec{\sigma})^n = \sum_n \frac{\theta^{2n}}{2n!} + i\sum_n \frac{(-1)^n\theta^{2n+1}}{(2n+1)!}\cdot(\vec{n}\vec{\sigma})$

$(\vec{n}\vec{\sigma})^{2n} = \mathbb{1}, \quad (\vec{n}\vec{\sigma})^{2n+1} = (\vec{n}\vec{\sigma}) \qquad = \cos(\theta)\,\mathbb{1} + i\sin(\theta)\cdot(\vec{n}\vec{\sigma})$

$$U(t) = \cos(\tilde{\Delta}t)\,\mathbb{1} + i\sin(\tilde{\Delta}t)\cdot\frac{1}{\tilde{\Omega}}\cdot\left[\frac{\delta}{2}\sigma^z + \sqrt{n}\, g\, \sigma^x\right]$$

$\hookrightarrow U(t)|g,n\rangle \sim \cos(\sqrt{n}gt)\,|g,n\rangle + i\sin(\sqrt{n}gt)\,|e,n+1\rangle$ when $\delta = 0$

P_g



$\sim \cos^2(\sqrt{n}gt) \Rightarrow$ Rabi frequency $\Omega = 2\sqrt{n}\,g$

t

Eigenenergies

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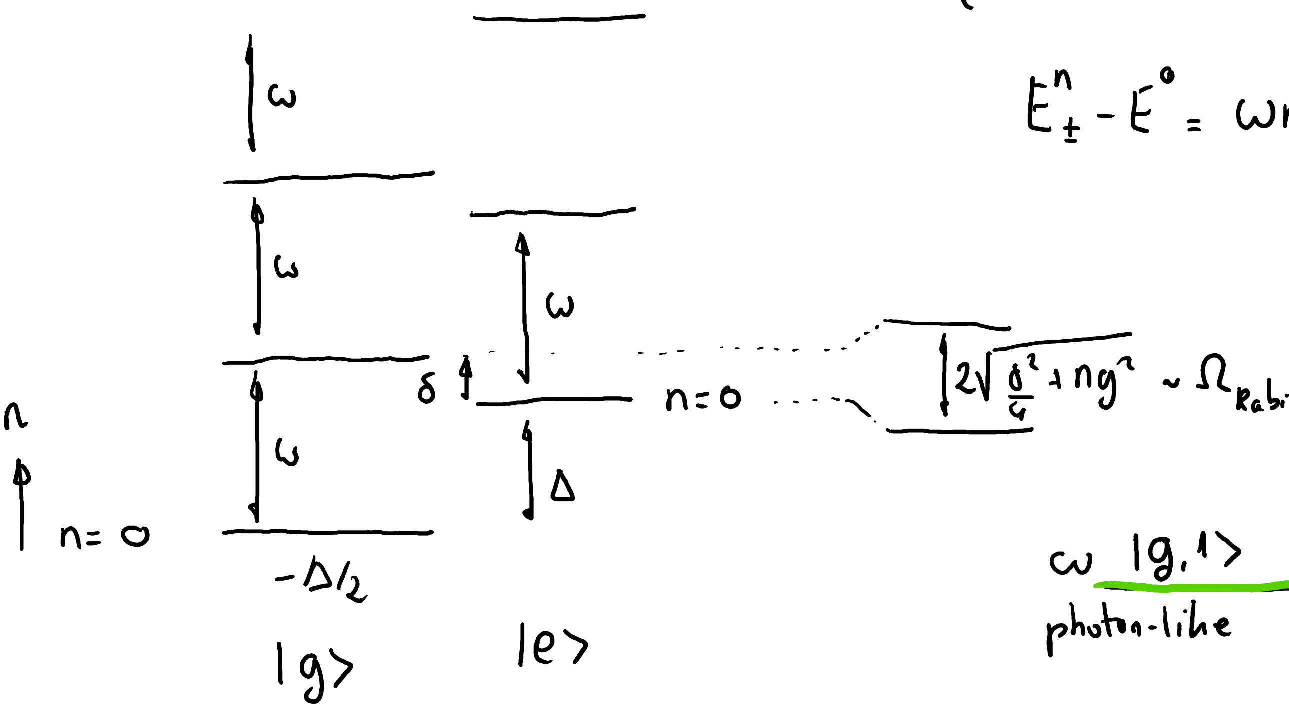
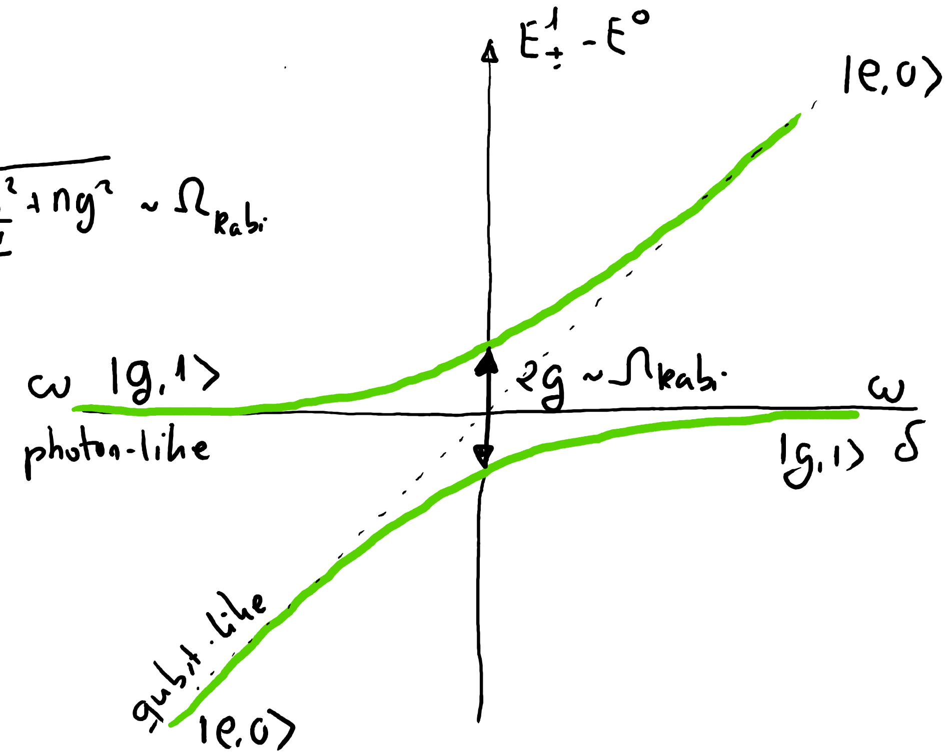
We can diagonalize each of the boxes

$$\begin{cases} E^0 = -\dfrac{\Delta}{2} \\ E_\pm^n = \omega(n-1/2) \pm \sqrt{\left(\dfrac{\Delta-\omega}{2}\right)^2 + n g^2} \end{cases}$$

$$E_\pm^n - E^0 = \omega n - \frac{\delta}{2} \pm \sqrt{\frac{\delta^2}{4} + n g^2}$$





Weak versus strong coupling

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$$\begin{cases} H = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon}{2}\sigma^x + g\sigma^x i(a^\dagger - a) + \omega a^\dagger a \\ \mathcal{L}[\rho] = \frac{\kappa}{2}\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right) + \frac{\gamma}{2}\left(2\sigma^-\rho\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-\right) \end{cases}$$

When coupling is weak, dynamics is overdamped: decay (κ, γ) happens well before any energy exchange (time $\sim 1/g$) may take place

$$g \ll \sqrt{\kappa\gamma} \Rightarrow \text{weak coupling}$$

We find experimentally

$$g \gg \sqrt{\kappa\gamma} \Rightarrow \text{strong coupling}$$

and qubit has time to talk to cavity

Non-Hermitian Hamiltonians

Thursday, February 18, 2016 11:26 AM

Goal: to study the effect of dissipation on the Rabi oscillations

Assumption: we start in a subspace $\mathcal{H}_N := \mathrm{lin}\{|e,N-1\rangle, |g,N\rangle\} = \underbrace{P_N}_{\#\text{ excit.}}^{\text{proyector}} \mathcal{H}$ ← all states

Realization: dissipation drains subspace $\mathcal{H}_j$ to feed $\mathcal{H}_{j-1}$. This is evident when we study the boxes of the dty matrix

$$\rho = \begin{pmatrix} \ddots & & & \\ & \rho_n & 0 & \\ & 0 & \rho_{n+1} & \\ & & & \ddots \end{pmatrix}$$

$$\rho_N \equiv P_N \rho P_N$$

$$\partial_t \rho_{N-1} = \frac{\kappa}{2}\left[2a\rho_N a^\dagger - a^\dagger a \rho_{N-1} - \rho_{N-1} a^\dagger a\right] + \frac{\gamma}{2}\left[2\sigma^-\rho_N\sigma^+ - \sigma^+\sigma^-\rho_{N-1} - \rho_{N-1}\sigma^+\sigma^-\right] - \frac{i}{\hbar}[H,\rho]$$

$$\left.\begin{array}{l} \mathrm{Op}\ N = n + \frac{1}{2}(\sigma^z+1) \\ \quad = \#\text{ excitations} \\ \text{Commutes with } H \end{array}\right\} \rightsquigarrow$$

Non-Hermitian Hamiltonians (2)

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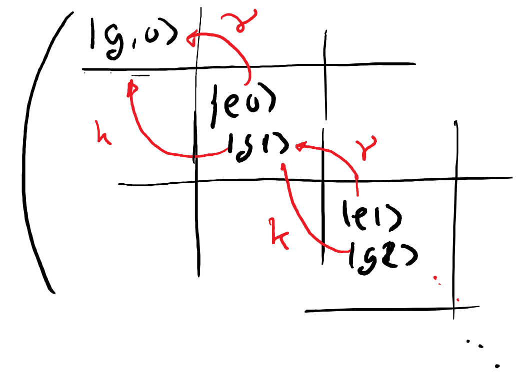
For $\rho(0) = \rho_N$ we have $\rho_{M>N} = 0$ and evolution is given by

$$\partial_t \rho_N(t) = -\frac{i}{\hbar}\left[H, \rho_N(t)\right] - \frac{\kappa}{2}\left(a^\dagger a \rho_N + \rho_N a^\dagger a\right) - \frac{\gamma}{2}\left(\sigma^+\sigma^- \rho_N + \rho_N \sigma^+\sigma^-\right)$$

$$\rho_N(t) = e^{-i\bar{H}t/\hbar}\, \rho_N(0)\, e^{+i\bar{H}^\dagger t/\hbar}$$



with $\bar{H} = H - \frac{i\hbar\kappa}{2} a^\dagger a - \frac{i\hbar\gamma}{2}\sigma^+\sigma^-$

So it is like evolution with a non-Hermitian operator

$$i\hbar\partial_t|\psi_N\rangle = \bar{H}|\psi_N\rangle \qquad \|\psi_N\|^2 = \mathrm{tr}(\rho_N(t)) \sim e^{-\kappa t}\, e^{-\gamma t}$$

$$\frac{\bar{H}}{\hbar} \sim \omega\left(N - \tfrac{1}{2}\right) - i\frac{\kappa}{2}(N-1) + \begin{array}{cc} \begin{pmatrix} \delta/2 - i\gamma/2 & \sqrt{n}\,g \\ \sqrt{n}\,g^* & -\delta/2 - i\kappa/2 \end{pmatrix} & \begin{array}{l} \leftarrow |e,N-1\rangle \\ \\ \leftarrow |g,N\rangle \end{array} \\ \begin{array}{cc} \uparrow \\ |e,N-1\rangle \end{array} \quad \begin{array}{cc} \uparrow \\ |g,N\rangle \end{array} & \end{array}$$

Resonant solution

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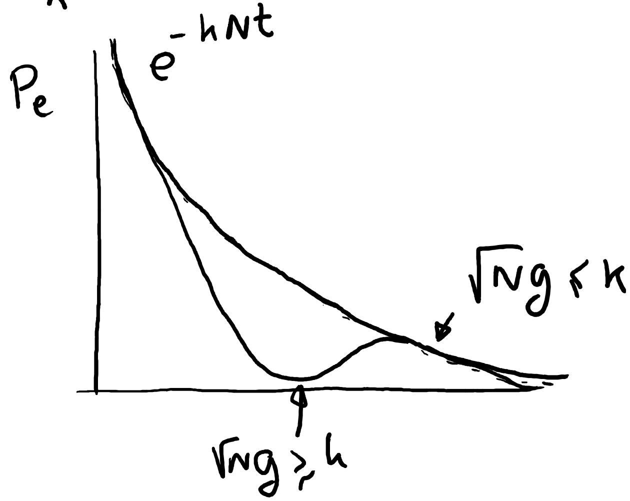
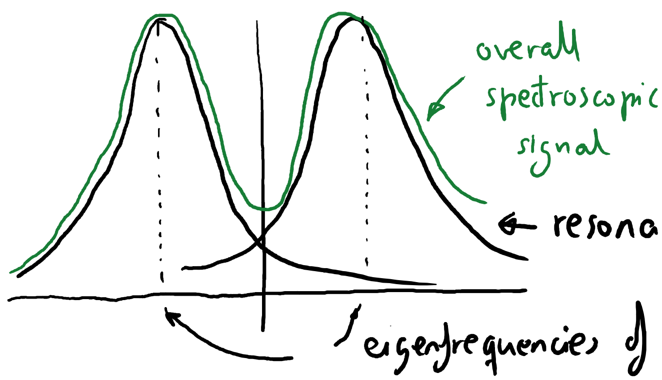
We focus on $\gamma = k$, $\omega = \Delta$, $\delta = 0$

$$\frac{\tilde{H}}{\hbar} \sim \omega N - i\frac{k}{2} N + \sqrt{N} g \sigma^x \rightarrow |\psi_N(t)\rangle \sim e^{-i\omega N t - ikNt/2}\left[\cos(\sqrt{N} g t) + i \sin(\sqrt{N} g t)\sigma^x\right]$$



- Oscillations become exponentially damped and unless $g > k/\sqrt{N}$, they cannot be detected
- These oscillations determine also the linewidth of the J-C eigenstates under external driving

$$\tilde{H} + \Omega(t)\sigma^x \quad \text{(qualitative picture)}$$



Spectroscopy

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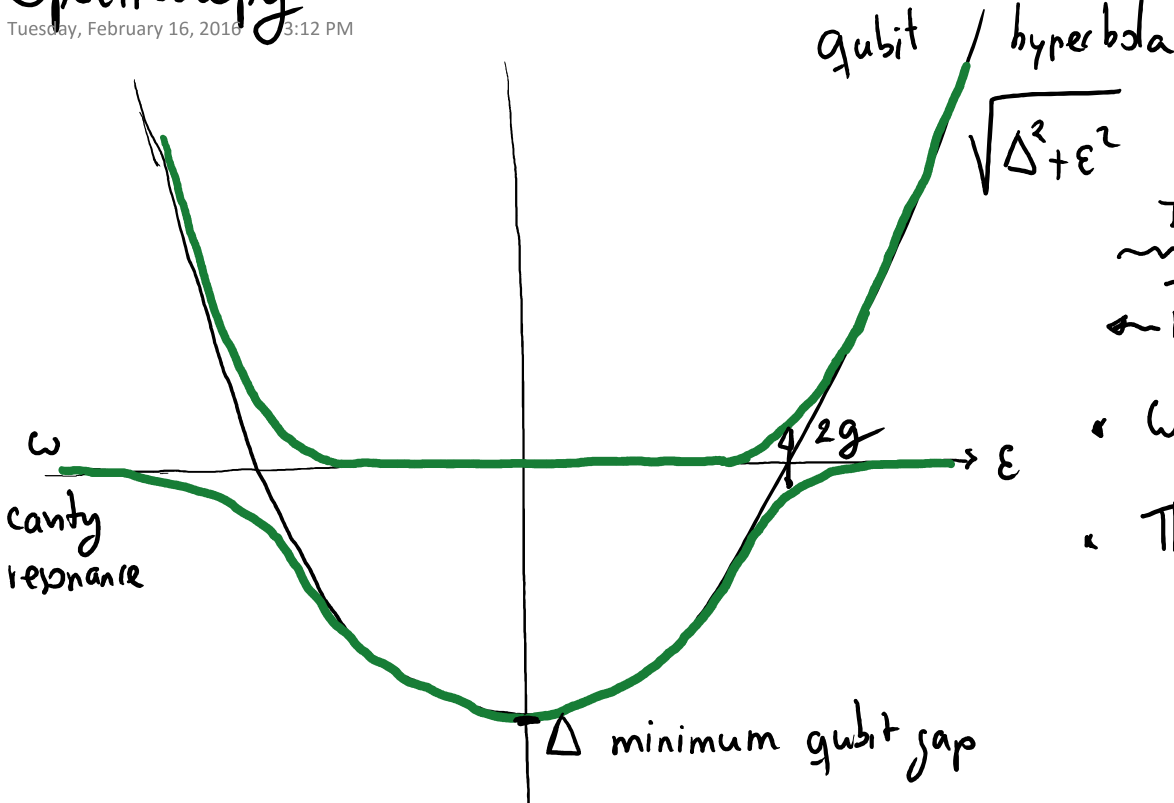

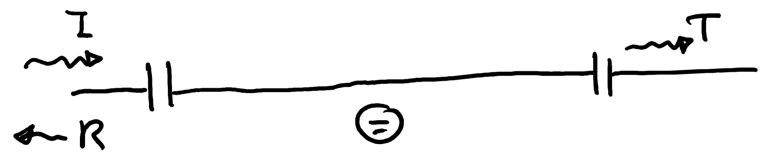

- We inject a time dependent signal $I(t)$
- The transmission depends on frequency

$$T(\omega) \sim \frac{1}{(\omega-\omega_{reson})^2+(\Gamma/2)^2}$$

where "ω_{reson}" is any resonance in our system and is given by the J-C model

- Γ is related to the coupling to the lines, or cavity emission rate (κ) $\rightarrow$ $\Gamma \sim \frac{\kappa}{2}+\frac{\gamma}{2}+\Gamma_\varphi+\dots$ plus all other sources of decoherence

Rabi vacuum splitting

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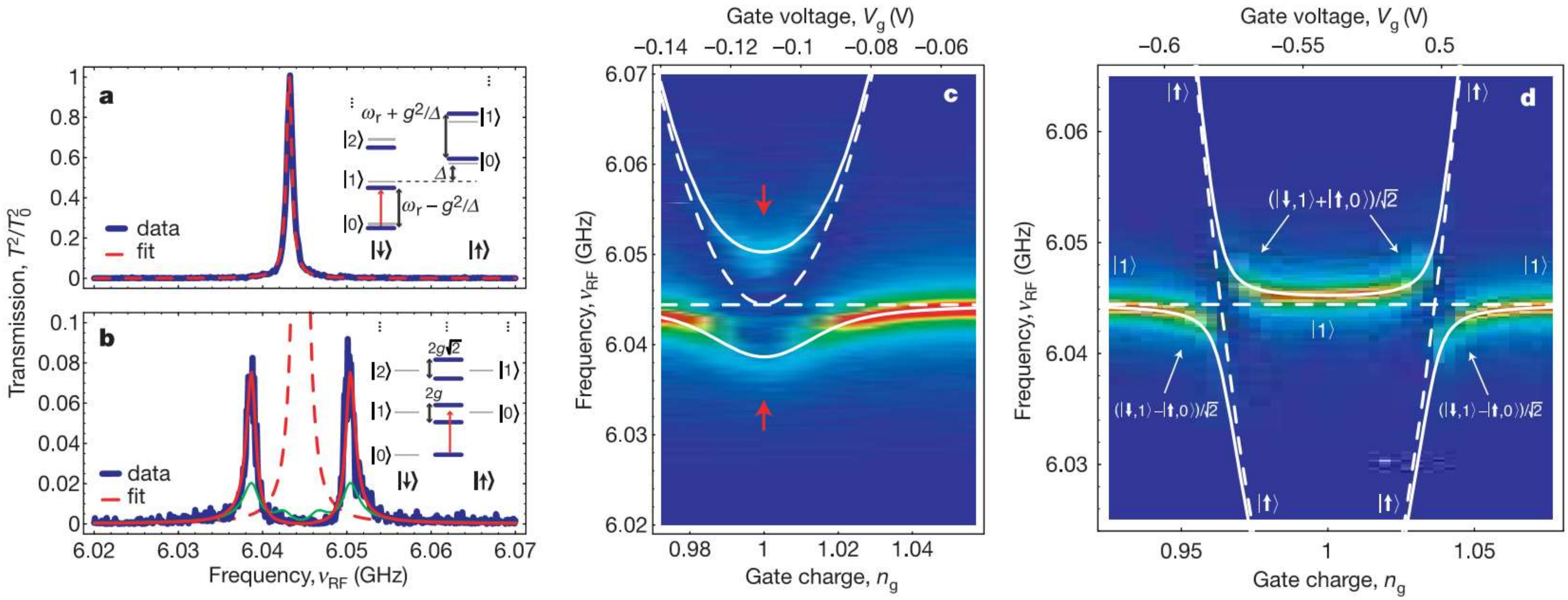
Dispersive

Resonant Rabi splitting



[**Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics**] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.- S. Huang, J. Majer, S. Kumar, S. M. Girvin and R. J. Schoelkopf *Nature* **431**, 162-167(9 September 2004)

Linewidths → κ, γ, n_{th} ...

$\kappa \sim 2\pi \cdot 0,8\,MHz$ $\quad n_{th} \lesssim 0.06$

$\gamma \sim 2\pi \cdot 0.7\,MHz$ $\quad \Omega_{Rabi} = 2\pi \cdot 11,6\,MHz$

Dispersive regime

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* This is a regime in which $\delta > g$. the qubit is far detuned from resonator

$$E_\pm^n = \omega n - \delta/2 \pm \sqrt{\frac{\delta^2}{4} + ng^2} \approx \omega n - \frac{\delta}{2} \pm \frac{\delta}{2}\left(1 + \frac{4ng^2}{\delta^2}\right)^{1/2}$$

$$\approx \omega n + \delta(\pm 1 - 1) \pm 2ng^2/\delta$$

$\delta = \Delta - \omega$

* This induces us to treat $\sqrt{n}\, g\, \sigma^x$ as a perturbation

$$H_n \sim \underbrace{\left(\omega n - \frac{\Delta}{2}\right)|g,n\rangle\langle g,n| + \left(\omega(n-1) + \frac{\Delta}{2}\right)|e,n-1\rangle\langle e,n-1|}_{H_0} + \underbrace{g\sqrt{n}\left(|e,n-1\rangle\langle g,n| + h.c\right)}_{H_1}$$

$$H_{eff}^n \sim H_0 + |g,n\rangle \frac{\langle g n | H_1 | e n-1\rangle\langle e n-1 | H_1 | g n\rangle}{E(g,n) - E(e,n-1)} \langle g,n| + (\text{other state})$$

$= -\Delta + \omega$

$$\sim \left(\omega n - \frac{\Delta}{2} - \frac{g^2 n}{\delta}\right)|g,n\rangle\langle g,n| + \left(\omega(n-1) + \frac{\Delta}{2} + \frac{g^2 n}{\delta}\right)|e,n-1\rangle\langle e,n-1|$$

notice sign difference

Dispersive Hamiltonian

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$(\delta = \Delta - \omega)$

$$H \sim \left(\omega n - \frac{\Delta}{2} - \frac{g^2 n}{\delta}\right)|g,n\rangle\langle g,n| + \left(\omega(n-1) + \frac{\Delta}{2} + \frac{g^2 n}{\delta}\right)|e,n-1\rangle\langle e,n-1|$$

$$\sim \omega a^\dagger a + \frac{\Delta}{2}\sigma^z - \frac{g^2}{\delta}\, a^\dagger a \underbrace{\left[\frac{-\sigma^z + 1}{2}\right]}_{|g\rangle\langle g|} + \frac{g^2}{\delta}\underbrace{(a^\dagger a + 1)}_{n}\underbrace{\left[\frac{\sigma^z + 1}{2}\right]}_{|e\rangle\langle e|}$$

$$\sim \underbrace{\left[\omega + \frac{g^2}{\delta}\sigma^z\right] a^\dagger a}_{\text{change in the resonator frequency depending on the qubit state}} + \underbrace{\left(\frac{\Delta}{2} + \frac{g^2}{\delta 2}\right)\sigma^z}_{\text{Renormalization of the qubit resonance frequency}}$$

$$|g\rangle \rightarrow \omega - \frac{g^2}{\delta}$$

$$|e\rangle \rightarrow \omega + g^2/\delta$$

Dispersive interaction

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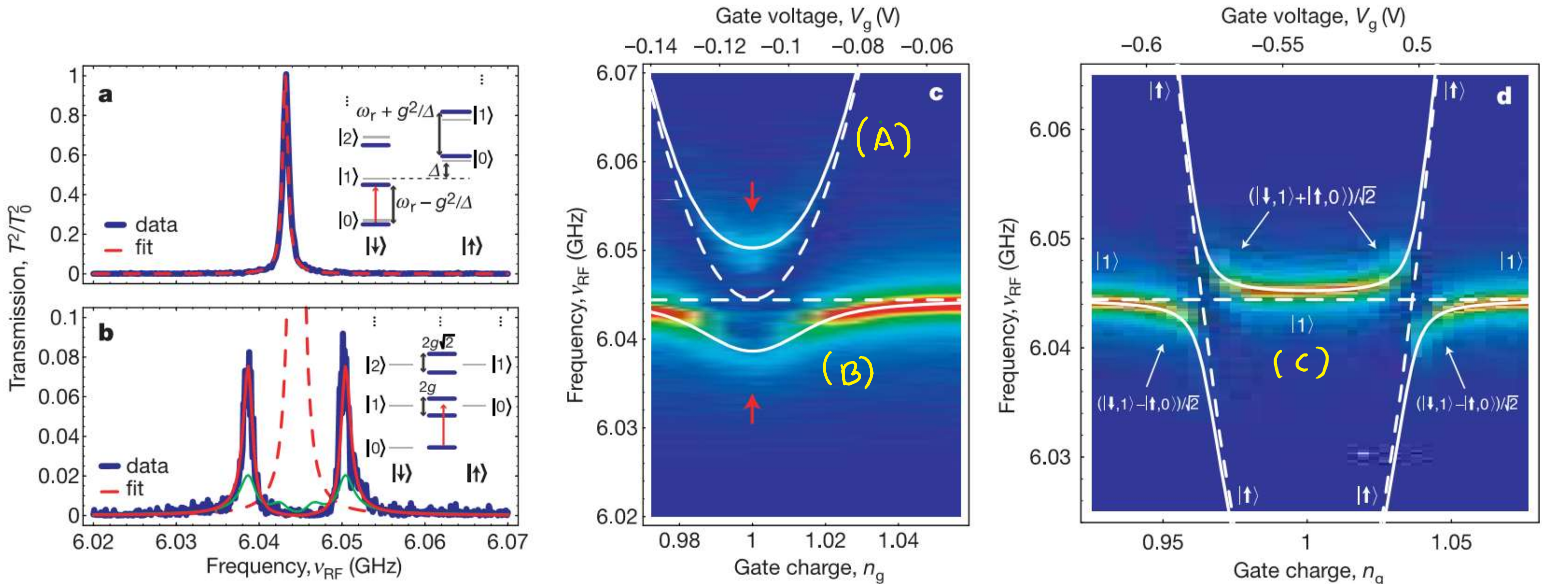

This figure contains two lines

↳ transitions $|g,0\rangle \leftrightarrow |e,0\rangle$ (A) with frequency $\Delta + \frac{g^2}{\delta} + \cancel{\frac{g^2}{\delta} a^\dagger a}$

↳ transitions $|g,n\rangle \leftrightarrow |g,n+1\rangle$ (B) with frequency $\left(\omega - \frac{g^2}{\delta}\right)$

In both, $\delta = \Delta - \omega > 0$

this figure shows a positive push of cavity (C) because $\delta < 0$

Different paradigms

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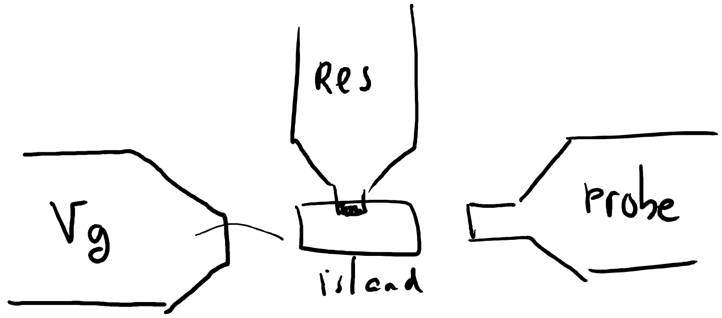
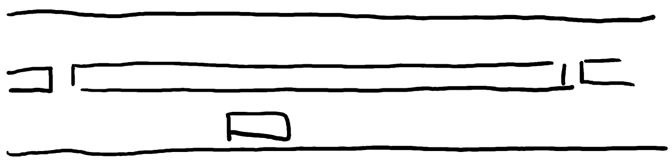
Before 2004



We connect the qubit to open cables and detectors

These devices induce decoherence because they behave as open baths themselves

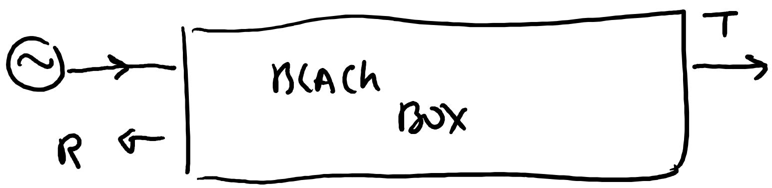
c.QED



Qubit is safely confined inside the resonator and it couples mostly to it

$$g \sim 10\text{-}100 \text{ MHz}$$

$$\gamma \lesssim 1 \text{ MHz}$$



Detuning the qubit protects it on a timescale $\sim \gamma^{-1}$

We can interrogate the combined system

Filtering & isolation

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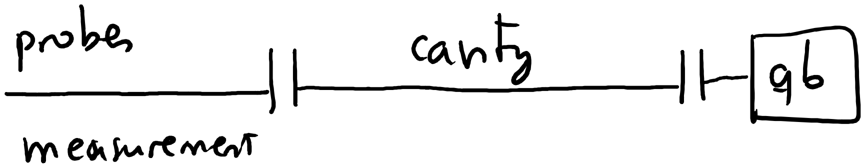

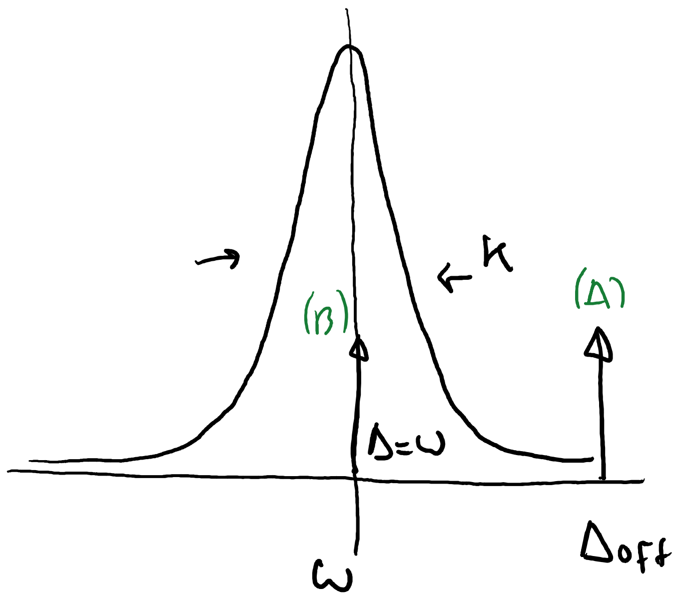

The cavity acts as a narrow filter that

1) Isolate the qubit
2) Only allows a small set of frequencies to pass through

When the qubit is off-resonant, is only weakly coupled to the bath (A)

$$J_{cav}(\omega) \sim \frac{J(\omega)_{cable}}{(\omega-\Delta)^2 + (\kappa/2)^2}$$

$$\gamma_{qb} \sim J_{cav}(\Delta) + \gamma_{intrinsic} \qquad \gamma_{intrinsic} \sim 1\,\mathrm{MHz}$$

(B) Bringing the qubit close to resonance, we speed up its dynamics

How to measure a quantum observable?

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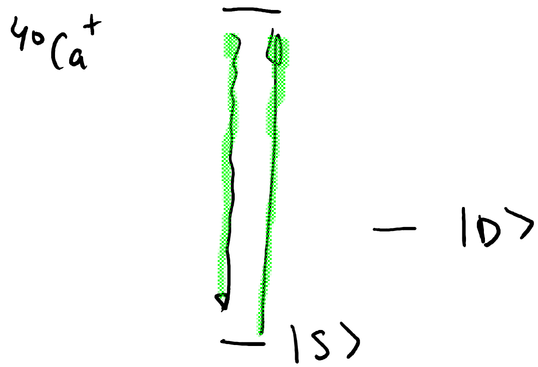
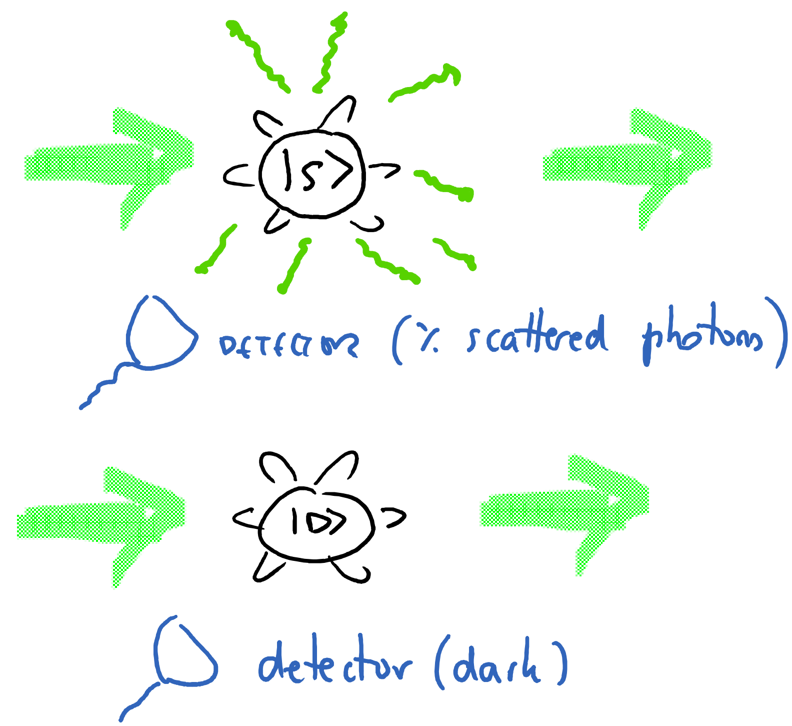
In trapped ions or ultracold atoms: fluorescence provides information about σ^z



The ion is excited with a laser that only can excite the $|S\rangle$ level, not $|D\rangle$

When it does, the atom reemits light in all directions



$|S\rangle \equiv |0\rangle \rightarrow$ fluorescence

$|D\rangle \equiv |1\rangle \rightarrow$ dark

What about the cavity?

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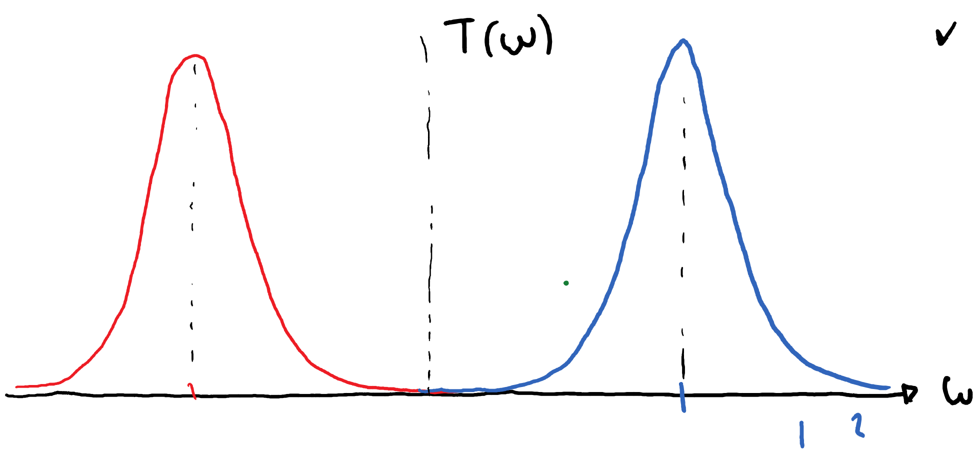
1) We don't have cyclic transitions but the resonances of the cavity are modified by the qubit

2) We do not want to work outside the dispersive regime because in any other point σ^z or $a^\dagger a$ are not well defined

Together: $H = \left(\omega + \frac{g^2}{\delta}\sigma^z\right)a^\dagger a + \frac{1}{2}\left(\Delta + \frac{g^2}{\delta}\right)\sigma^z$ has two cavity resonances

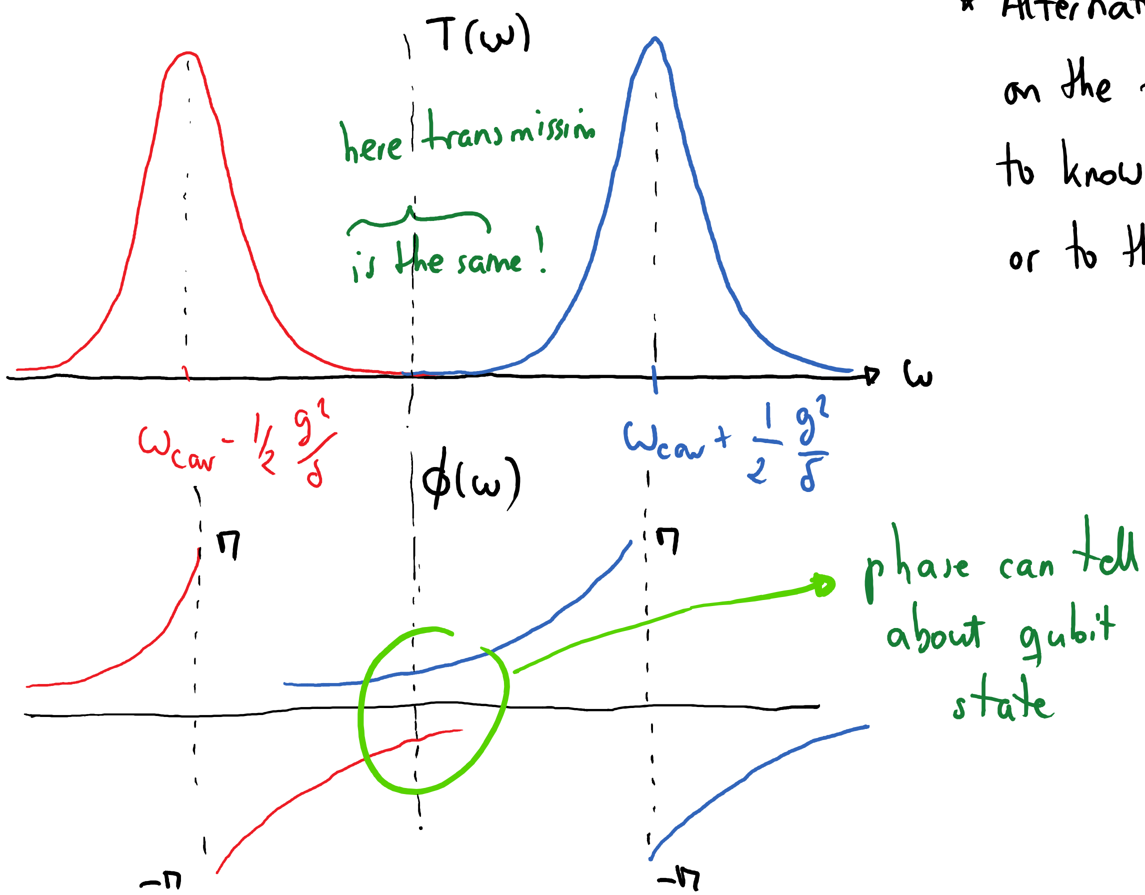


✓ We could drive the cavity at $\left(\omega + \frac{g^2}{\delta}\right)$ and if there is transmission we know the qubit was in $|e\rangle$

× Problem: residual transmission from qubit being $|g\rangle$

QND measurement of the qubit

Tuesday, February 16, 2016 4:04 PM



* Alternative: use the phase imprinted on the transmitted or reflected light to know whether we are to the left or to the right of a resonance

$$T \sim \frac{\sqrt{\kappa}}{(\omega - \omega_{reson}) + i\kappa/2}$$

$$\arg(T) \sim \pi + (\omega - \omega_{reson}) \ldots$$

* Another advantage: because the cavity is only weakly populated, state of qubit is almost not distorted

* Projective measurement: after the interaction w. the field, even if we do not measure it, the qubit ends up in $|e\rangle$ or $|g\rangle$ with the appropriate probability