Reminder

Tuesday, February 16, 2016 3:46 PM

We have reached a model of a qubit

$$H = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon}{2}\sigma^x$$

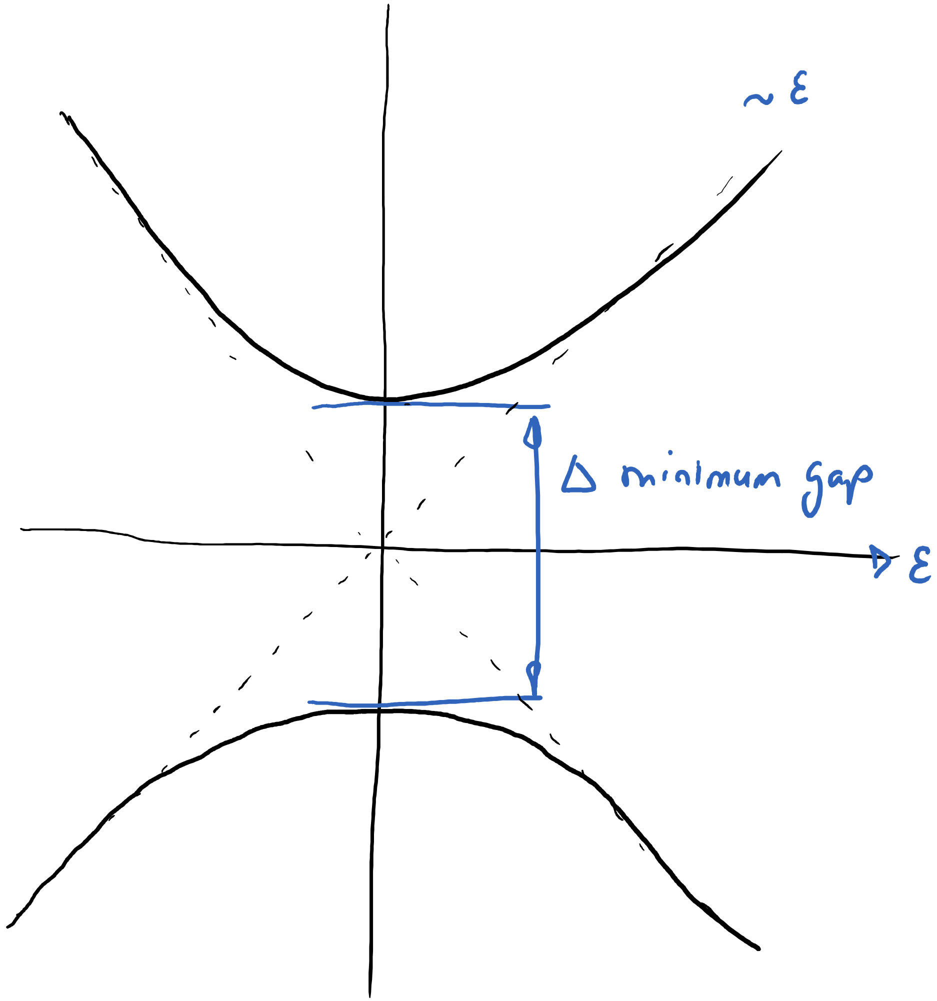
$\sigma^z = |1\rangle\langle 0| + |0\rangle\langle 1|$ ~ tunelling

$\Delta \sim E_J$ Josepson energy

$\sigma^x = |1\rangle\langle 1| - |0\rangle\langle 0|$ ~ charge

$\varepsilon \sim V_g$ external voltage

but the considerations that follow are general



$E_{\pm} = \pm\frac{1}{2}\sqrt{\Delta^2 + \varepsilon^2}$, qubit hyperbola

Qubit control

Saturday, February 13, 2016 3:43 PM

a) Cooling : set up a field and wait for the qubit to relax

$$\omega \sim 2\text{-}9 \text{ GHz} \gg k_B T$$

$$P_e \sim \exp\left(-\hbar\omega / k_B T\right) \ll 1$$

b) Adiabatic passage : move from one ground state to another by slowly changin $\begin{cases} E_J \\ \varepsilon \end{cases}$

↳ If $\hbar\Delta(t)$ is the instantaneous gap,

$v = \frac{1}{\Delta} \cdot \frac{d\Delta}{dt}$ the speed

$\tau \sim \frac{1}{v}$ the time scale for change

$\tau \gg \frac{1}{\Delta} \Rightarrow$ adiabatic condition

↳ $H = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon}{2}\sigma^x, \Rightarrow \frac{\dot{\varepsilon}(t)}{\varepsilon} \ll \Delta$

c) Diabatic or instantaneous change : just the opposite! $\tau \ll \frac{1}{\min(\varepsilon, \Delta)}$

min (g cp)

Qubit control (2)

Saturday, February 13, 2016 4:00 PM

d) Free evolution : $H = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon}{2}\sigma^x$ generates some rotation

$i\frac{d}{dt}|\psi\rangle = H|\psi\rangle \Rightarrow |\psi(t)\rangle = e^{-iH(\Delta,\varepsilon)t/\hbar}|\psi(0)\rangle$

↳ This does not include all single-qubit rotations $U = \exp(i\vec{n}\cdot\vec{\sigma}\cdot\theta)$

↳ They can only be obtained by composing gates

↳ Only σ^z is obtained exactly ($\varepsilon = 0$)

↳ σ^x rotations only approximate $(|\varepsilon| \gg |\Delta|) \Rightarrow$ too large fields enhance decoherence!

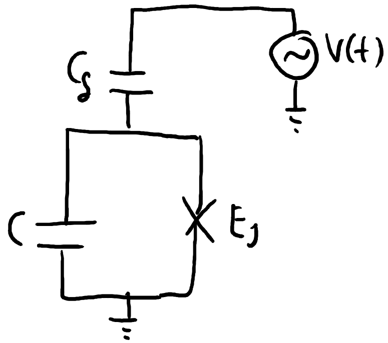
e) External driving : make Δ or ε depend on time to engineer arbitrary $U(t)$

End of lecture Feb 15th

Qubit control (3)

Saturday, February 13, 2016 5:21 PM

$$H = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon(t)}{2}\sigma^x$$



* Let us assume a sinusoidal driving of the potential

$$H = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon_0}{2}\cos(\omega t+\phi)\,\sigma^x = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon(t)}{2}\sigma^x$$

* We start by separating free evolution

$$i\frac{d}{dt}U = \left[H_0 + \frac{\varepsilon(t)}{2}\sigma^x\right]U$$

$$U(t) = \exp(-iH_0 t)\,W(t), \quad W(0) = \mathbb{1}$$

$$i\frac{dW}{dt} = e^{iH_0 t}\frac{\varepsilon(t)}{2}\sigma^x e^{-iH_0 t}\cdot W = H_I(t)\,W(t)$$

* The interaction term contains various rotations

$$H_I(t) = \frac{\varepsilon_0}{2}\left[e^{i\omega t + i\phi} + e^{-i\omega t - i\phi_0}\right]\left[\sigma^+ e^{i\Delta t} + \sigma^- e^{-i\Delta t}\right]$$

$$\simeq \frac{\varepsilon_0}{2}\left[e^{i(\omega-\Delta)t + i\phi}\sigma^- + e^{-i(\omega-\Delta)t - i\phi}\sigma^+\right] + (\text{oscillations w. } \omega+\Delta)$$

* This suggests keeping only the rotating terms and redoing the transf. with $H_0 = \frac{\omega}{2}\sigma^z$

$$U(t) \simeq e^{-i\omega t\sigma^z/2}\, e^{-iH_{RWA}t} \qquad H_{RWA} = \frac{\Delta-\omega}{2}\sigma^z + \frac{\varepsilon_0}{2}\sigma^x$$

Rotating Wave Approximation (RWA)

Rotating wave

Saturday, February 13, 2016 5:42 PM

* We focus on wavefunctions $|\psi(t)\rangle = \psi_0|0\rangle + \psi_1|1\rangle$

* We introduce the rotating frame as before $|\psi(t)\rangle = e^{-iH_0 t}|\phi(t)\rangle$ $H_0 = \frac{\omega}{2}\sigma^z$

$$i\frac{d}{dt}|\phi(t)\rangle = \Big[\underbrace{\frac{\Delta-\omega}{2}\sigma^z + \frac{\varepsilon_0}{2}\sigma^x + \frac{\varepsilon_0}{2}\big[e^{i2\omega t+i\phi}\sigma^+ + h.c}_{H_2(t)}\big]\Big]|\phi(t)\rangle$$

* Magnus expansion

$$i\frac{d}{dt}U = H_2(t)\,U \Rightarrow U(t) = \exp\left(-i\,\Sigma\,\bar{H}_n(t)\right)$$

$$\bar{H}_1(t) = \int_0^t d\tau_1\, H_2(\tau)$$

$$\bar{H}_2(t) = \frac{-i}{2\hbar}\int_0^t d\tau_1 \int_0^{\tau_1}[H_2(\tau_1), H_2(\tau_2)]\,d\tau_2$$

$\vdots$

$$\bar{H}_1(t) \sim H_{RWA}\cdot t + \underbrace{\int_0^t e^{i2\omega t+i\phi}\frac{\varepsilon_0}{2}(\dots)}_{\mathcal{O}\left(\frac{\varepsilon_0}{2\omega}\right)}$$

Higher order corrections possible...

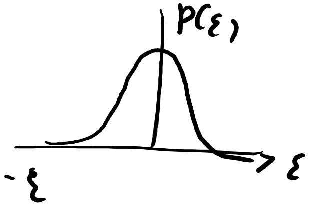
↳ non-rotating terms at $\varepsilon_0^2, \varepsilon_0^4 \ldots$

Pure dephasing

Sunday, February 14, 2016 1:36 PM

* A dephasing source can be any fluctuating field that is diagonal in the qubit basis, it is Markovian and it shifts the energy levels of the qubit randomly around

$H = \frac{1}{2}(\Delta+\varepsilon)\sigma^z$ $\quad p(\varepsilon)$ = probability of certain "shift" ε.

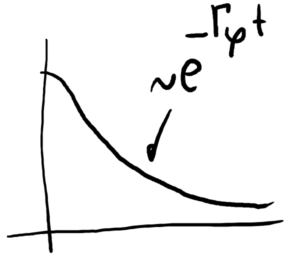


* In the rotating frame w. Δ we can integrate $\rho(t)$

$$\rho(t) \approx \int_0^{\Delta t} d\varepsilon\, p(\varepsilon)\, e^{-i\varepsilon t\sigma^z/2}\,\rho(0)\,e^{i\varepsilon t\sigma^z/2}$$

$$\approx \begin{pmatrix} \rho_{00} & \rho_{10} f(t) \\ \rho_{10} f^*(t) & \rho_{11} \end{pmatrix}$$

$$f(t) = \int_0^{\Delta t} d\varepsilon\, p(\varepsilon)\, e^{-i\varepsilon t}$$



Pure dephasing (2)

Tuesday, February 16, 2016 8:52 PM

P. ej. $p(\varepsilon) = \frac{\gamma_\phi}{\pi} \frac{1}{\varepsilon^2 + \gamma_\phi^2} \Rightarrow J(t) = e^{-t\gamma_\phi}$

More generally

$$\rho(t+\delta t) \sim \rho(t) - \delta t \cdot \gamma_\phi \begin{pmatrix} 0 & \rho_{01} \\ \rho_{10} & 0 \end{pmatrix} \sim \rho(t) + \frac{\delta t}{2} \gamma_\phi \left[\sigma^z \rho \sigma^z - \rho \right]$$

At short times $\partial_t \rho \sim \frac{\Gamma_\varphi}{2} \left[\sigma^z \rho \sigma^z - \rho \right]$
$(\Delta t \to 0)$

$\begin{cases} \Gamma_\varphi = \text{dephasing rate } (s^{-1}) \\ T_2 = \frac{1}{\Gamma_\varphi} = \text{dephasing time or T-2 } (s) \end{cases}$

Master equation

Saturday, February 13, 2016 6:41 PM

Straight-forward quantum optics model

1) $H = \underbrace{H_{qubit} + H_{bath}}_{H_0} + \underbrace{H_{int}}_{H_1}$ $\qquad H_{int} \sim \sum_\omega \theta_{qb} \left(g_\omega A_\omega + H.c.\right)$

2) Assume quasistationary bath $\rho \approx \rho_{qb} \otimes \rho_{bath}(T)$ (T: temperature)

3) Derive an effective equation for ρ_{qb}

a) Move to an interaction picture $\rho(t) = e^{-iH_0 t} \overbrace{\left(\bar{\rho}_{qb} \otimes \bar{\rho}_b\right)}^{\bar{\rho}} e^{iH_0 t}$

b) Develop equation for $\bar{\rho}(t)$ up to 2^{nd} order in $H_{int}(t) = e^{+iH_0 t} H_{int} e^{-iH_0 t}$

c) Impose $\bar{\rho} = \bar{\rho}_{qb} \otimes \bar{\rho}_{bath}$ and trace over $\bar{\rho}_{bath}$ $\Rightarrow$ integro-diff. eq. for ρ_{qb}

d) Impose Markovian limit $\Rightarrow$ integral kernel becomes local in time

$\Rightarrow$ ordinary dif. eq. for $\bar{\rho}_{qb}$

Master equation (2) (see Milburn & Walls, Q. Optics)

Monday, February 15, 2016 8:21 PM

* We start by moving into the interaction picture
  - From $i\hbar\partial_t\rho = [H_0 + H_1, \rho]$ we split $\rho(t) = e^{-iH_0 t}\rho_I(t)\, e^{iH_0 t}$

    $i\hbar\partial_t \rho_I = [H_I(t), \rho_I]$, $H_I = e^{iH_0 t} H_1 e^{-iH_0 t}$

* We then use this equation recursively

$$\rho(t+\delta t) = \rho(t) - \frac{i}{\hbar}\int_t^{t+\delta t} [H_I(\tau), \rho(t)]\,d\tau + \left(-\frac{i}{\hbar}\right)^2 \int_t^{t+\delta t} d\tau_1 \int_0^{\tau_1} d\tau_2\, [H_I(\tau_1), [H_I(\tau_2), \rho(t)]] + \dots$$

* We introduce the quasistationary bath approx. $\rho(t) = \underbrace{\rho_s(t)}_{\text{system}} \otimes \underbrace{\rho_B}_{\text{bath}}$

  and impose $\mathrm{tr}\left(H_I(\tau)\,(\text{anything} \otimes \rho_B)\right) = 0$

$$\rho(t+\delta t) \approx \rho_s(t) + \left(\frac{-i}{\hbar}\right)^2 \int_t^{t+\delta t} d\tau_1 \int_t^{\tau_1} \mathrm{tr}_{BATH}\left([H_I(\tau_1), [H_I(\tau_2), \rho_s(t)\otimes\rho_B]]\right) d\tau$$

Master equation (3)

Tuesday, February 16, 2016 2:12 PM

* We will use the particular form of H_1 to prove that the integral can be written as a time-independent superoperator acting on $\rho_s(\tau)$

$$\rho(t+\delta t) = \rho_s(t) + \delta t \cdot \mathcal{L}[\rho_s(t)]$$

$$\Downarrow$$

$$\partial_t \rho_s = \mathcal{L}[\rho_s]$$

* Our qubit or "system" will be coupled linearly to a bath of bosons

$$H_1 \sim \sum_n g_n \underset{\text{qubit}}{A^+} \underset{\text{bath}}{B_n} + \text{H.c.}$$

Note that writing this will imply some rotating wave approximation

Master equation (4)

Monday, February 15, 2016 8:40 PM

- In the interaction picture we will have

$$H_I(t) = \sum_k g_k A^+ B_k e^{i(\Delta - \omega_k)t} + H_c$$

where "Δ" and "ω_k" are the system and bath eigenfrequencies

- Using this in the master equation leads to

$$\Delta \rho_s = \int_t^{t+\delta\tau} \left\{ I_{B^+B^+} V_{AA} + I_{BB^+} V_{A^+A} + I_{B^+B} V_{AA^+} + I_{BB} V_{A^+A^+} + H.c \right] d\tau \approx \delta t . \mathcal{L}[\rho]$$

to be proven

we need to show I, V are t. independent

$$V_{XY} = X \rho_s Y - Y X \rho_s$$

$$I_{B^+B} = \frac{1}{\hbar^2} \sum_{pq} g_p^* g_q \int_t^{\tau_1} d\tau_2 \langle B_p^+ B_q \rangle e^{+i(\omega_p - \Delta)\tau_1 - i(\omega_q - \Delta)\tau_2}$$

$$I_{BB^+} = \frac{1}{\hbar^2} \sum_{pq} g_p g_q^* \int_t^{\tau_1} d\tau_2 \langle B_p B_q^+ \rangle e^{-i(\omega_p - \Delta)\tau_1 - i(\omega_q - \Delta)\tau_2}$$

... and other terms

$\vdots$

Master equation (5)

Tuesday, February 16, 2016 2:14 PM

* We introduce the spectral function

$$J(\omega) = \sum_p 2\pi \frac{|g_p|^2}{\hbar^2} \delta(\omega - \omega_p)$$

and assume a thermal bath

$$\langle B_p B_q \rangle = \langle B_p^\dagger B_q^\dagger \rangle = 0$$

$$\langle B_p^\dagger B_q \rangle = n(\omega_p)\, \delta_{pq} \quad , \quad \langle B_q B_p^\dagger \rangle = (n(\omega_p) + 1)\, \delta_{pq}$$

dty of states

$$\left\{ n(\omega) \sim \text{thermal photon occupation \#} \sim \frac{\rho(\omega)}{e^{\hbar\omega/k_B T} + 1} \right.$$

so that for instance

$$I_{BB^\dagger} = \frac{1}{2\pi} \int_t^{\tau_1} d\tau_2 \int d\omega\, J(\omega) (n(\omega) + 1)\, e^{i(\omega - \Delta)(\tau_1 - \tau_2)}$$

$$= \frac{1}{2\pi} \int_t^{\tau_1} d\tau_2\; k(\tau_2 - \tau_1)$$

Master equation (6)

Tuesday, February 16, 2016 2:15 PM

* We assume a coarse grained integration $\delta\tau \gg T$ to effectively take the upper limit $(\tau_1 \to \infty)$

neglected

$$\int_0^{+\infty} d\tau_2 \, e^{i\varepsilon\tau_2} = \pi\delta(\varepsilon) - i\,\cancel{PV\left(\frac{1}{\varepsilon}\right)}$$

$$I_{B^+B} \approx \frac{1}{2\pi}\int d\omega \, J(\omega)\, n(\omega)\, \pi\delta(\omega-\Delta)\, e^{i(\omega-\Delta)\tau_1} = \frac{1}{2} J(\Delta)\, n(\Delta) \quad (\in \mathbb{R})$$

* Finally

$$\partial_t \rho = \frac{\gamma}{2}\left[2A\rho A^+ - A^+A\rho - \rho A^+A\right] \cdot (n_{th}(\Delta)+1)$$

$$+ \frac{\gamma}{2}\left[2A^+\rho A - AA^+\rho - \rho AA^+\right](n_{th}(\Delta))$$

$$\boxed{\gamma = J(\Delta)}$$ ← very important, will appear again and again

Pure dissipation for a qubit

Sunday, February 14, 2016 1:23 PM

Remember that we are rotating at freq Δ

The master equation in this case reads

$$\partial_t \rho_{gb} = \frac{\Gamma_1}{2}\left(2\sigma^- \rho_{gb}\sigma^+ - \sigma^+\sigma^-\rho_{gb} - \rho_{gb}\sigma^+\sigma^-\right) \equiv \mathcal{L}[\rho]$$

$\mathcal{L}[\bar{\rho}]$ is a Linbladian, a super-operator mapping matrices to matrices

$$\mathcal{L} : \mathbb{C}^{2\times 2} \to \mathbb{C}^{2\times 2}$$

Note that all valid master equations have Lindblad form $\mathcal{L}[\rho] = \sum_n A_n \rho A_n^\dagger$

↳ this is required for it to be a valid eq. for a dty matrix.

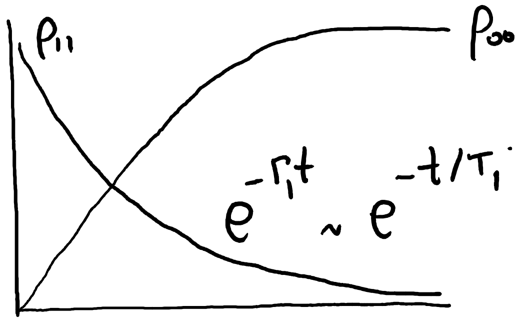
Can we solve it? Let's expand $\rho = \sum_{ij} \rho_{ij} |i\rangle\langle j|$ + constraint $\rho_{00} + \rho_{11} = 1$

Bloch equations (T1)

Sunday, February 14, 2016 1:30 PM

$$\partial_t \rho = \mathcal{L}[\rho] \Rightarrow \partial_t \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = \frac{\Gamma}{2} \begin{pmatrix} 2\rho_{11} & -\rho_{01} \\ -\rho_{10} & -2\rho_{11} \end{pmatrix}$$

$$\rho(t) = \begin{pmatrix} \rho_{00}(0) + (1-e^{-\Gamma_1 t})\,\rho_{11}(0) & \exp(-\Gamma_1/2\, t)\,\rho_{01}(0) \\ \exp(-\Gamma_1/2)\,\rho_{10}(0) & \exp(-\Gamma_1 t)\,\rho_{11}(0) \end{pmatrix}$$



$\Gamma_1 \equiv$ spontaneous emission rate (atoms) (s^{-1})
or decay rate (qubits)

$T_1 \equiv \dfrac{1}{\Gamma_1} =$ T-1 time (s)

$\Gamma_\varphi \equiv \dfrac{\Gamma_1}{2} =$ "dephasing rate" or rate of loss of coherence (s^{-1})

$T_2 = \dfrac{1}{\Gamma_\varphi} =$ T-2 time (s) or dephasing time

A slightly more rigorous approach ?

Tuesday, February 16, 2016 1:57 PM

* We consider our qubit coupled to a bosonic bath w. an Ohmic environment

$$H = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon}{2}\sigma^x + \sigma^x \sum_k \left(g_k a_k^\dagger - g_k^* a_k \right) + \sum_k \omega_k a_k^\dagger a_k$$

$$J(\omega) = 2\pi \sum_k \frac{|g_k|^2}{\hbar^2}\delta(\omega - \omega_k) \sim \pi \cdot \alpha$$

* This model can be integrated analitically using path integral, and showing dephasing (T_φ) and dissipation (T_1) through expectation values

$$\langle \sigma^\pm \rangle = e^{\pm i \tilde{\Delta} t} \quad e^{-t/t_\varphi}$$

$$\langle \sigma^z \rangle = \langle \sigma^z \rangle_\infty + \left[\langle \sigma^z \rangle_0 - \langle \sigma^z \rangle_\infty \right] e^{-t/T_1}$$

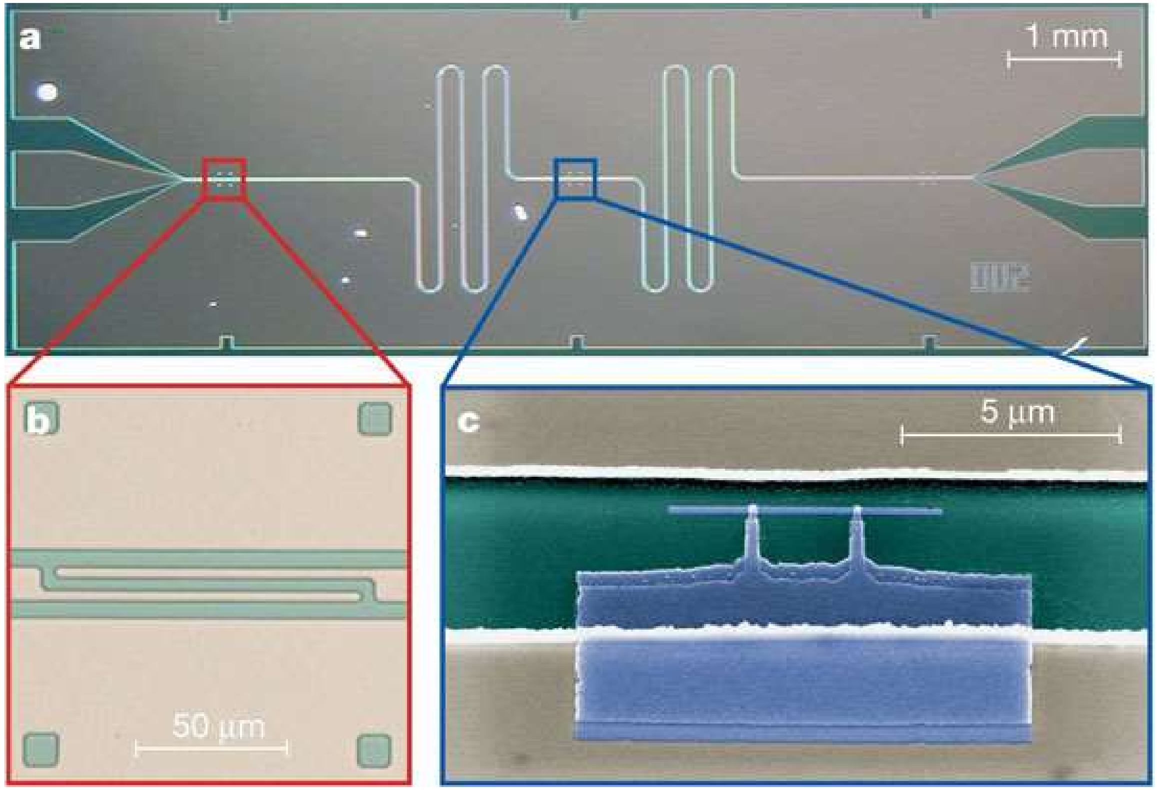
$$T_1^{-1} = \pi\alpha \left(\sin^2\eta\right) \tilde{\Delta} \coth \frac{\hbar\tilde{\Delta}}{k_B T} \longleftarrow \begin{cases} \Delta = \tilde{\Delta}\sin\eta \\ \varepsilon = \tilde{\Delta}\cos\eta \end{cases}$$

[illegible]

$$T_\varphi^{-1} = \frac{1}{2} T_1^{-1} + \pi \alpha \left(\cos^2 \eta \right) \cdot \frac{2 k_B T}{\hbar}$$

Circuit-QED setup

Thursday, February 11, 2016 9:29 AM



Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.- S. Huang, J. Majer, S. Kumar, S. M. Girvin and R. J. Schoelkopf *Nature* **431**, 162-167(9 September 2004)

* Things we expect

1) A microwave resonator that can be probed through transmission exp'ts

$$H_r = \hbar \omega a^\dagger a \sim \frac{1}{2C} q^2 + \frac{1}{2L} \phi^2$$

2) A charge qubit that is long lived an has tunable energy

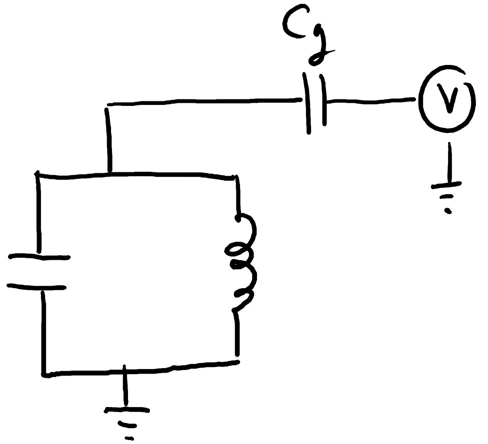
$$H_{qb} = \frac{\Delta(\phi)}{2} \sigma^z$$

3) A coupling bw. the charge of the qubit and the resonator voltage

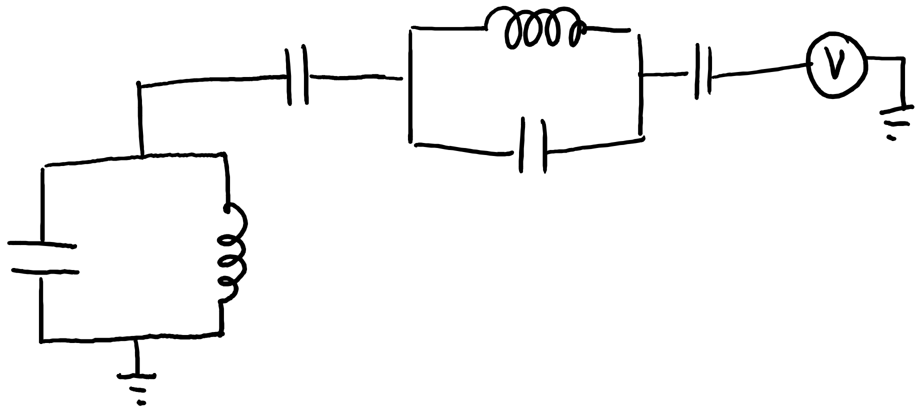
$$H_{int} \sim Q_{qb} \cdot V_r$$

linear electrical (dipole) coupling

Thursday, February 11, 2016 9:32 AM



$\Rightarrow$

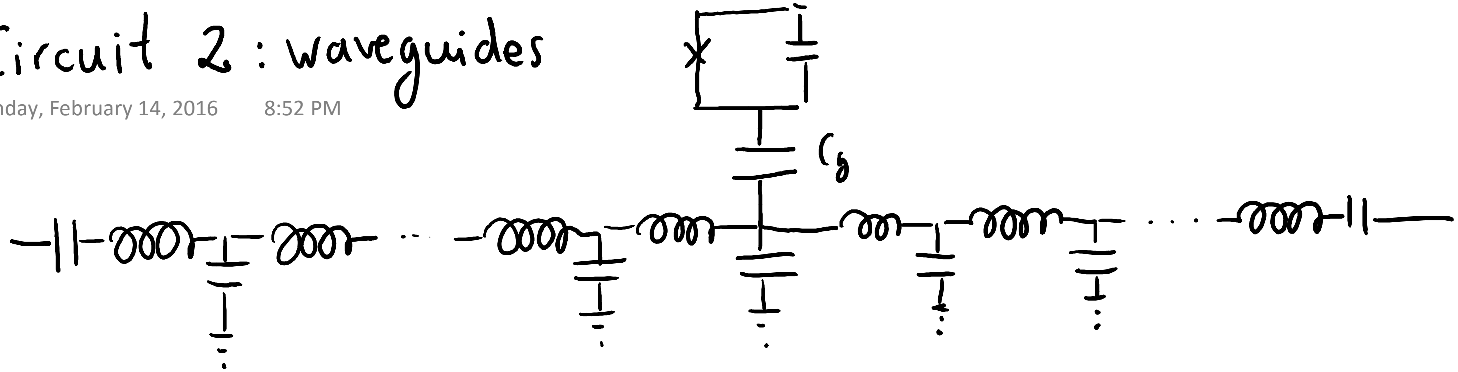


$$H \sim \frac{1}{2C_\Sigma}(q - q_g)^2 - E_J \cos(\varphi) \quad \Rightarrow \quad q_g = \frac{1}{C_g}\left(V_0 + \partial_t \phi_{LC}\right) \Rightarrow \text{linear coupling}$$

Usually works study just the influence of an external potential V and assume that this potential now includes the quantum fluctuations of the resonator $\sim \sum_n \sqrt{\frac{\hbar Z}{\ell k_n}} \, \omega_n \frac{i}{\sqrt{2}} (a_n^\dagger - a_n) = \partial_t \phi_{LC}(x_{qb})$

Circuit 2: waveguides

Sunday, February 14, 2016 8:52 PM



The usual approach is to take the previous result

$$H = \frac{1}{2C_\Sigma} q^2 - E_J \cos\left(\phi_1/\phi_0 \, 2\pi\right) + \int \left[\frac{1}{2C_0} \rho^2 + \frac{1}{2L_0} (\partial_x \phi)^2 \right] dx$$

$$+ \frac{C_g}{C_\Sigma} \cdot q \left[\frac{1}{C_0} \rho(x) + V_e \right] \Rightarrow \frac{C_g}{C_\Sigma} q \left[\sum_n \frac{\phi_n(x)}{\sqrt{\ell}} \sqrt{\frac{\hbar k_n}{Z}} \frac{i}{\sqrt{2}} (a^\dagger - a) + V_e \right]$$

We already saw that we can reach $\frac{1}{C_0} \rho \sim \mu V$

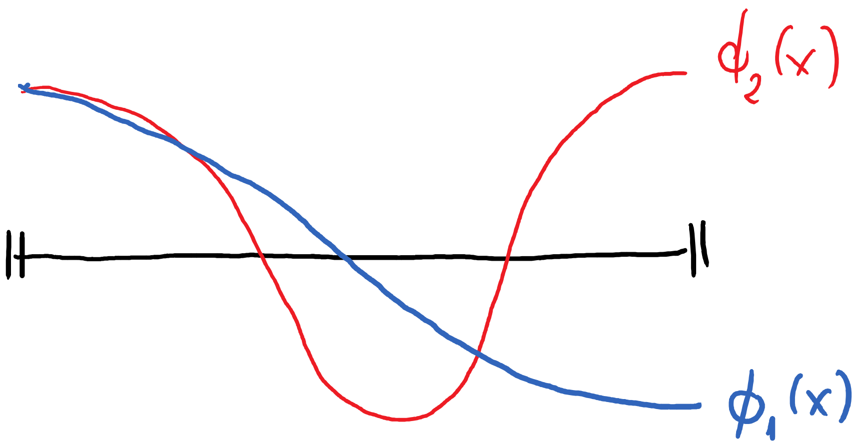
so an upper-bound is $\sim \mu eV$ $\left(C_g / C_\Sigma \sim 1/2, \; q \sim 2e \right)$ $\Rightarrow$ hundreds of MHz coupling

Coupling dependence

Tuesday, February 16, 2016 9:30 PM

$$\frac{C_g}{C_\Sigma} g \left[\sum_n \frac{\phi_n(x)}{\sqrt{\ell}} \sqrt{\frac{\hbar k_n}{Z}} \frac{i}{\sqrt{2}}(a^\dagger - a) + V_e \right]$$

1) The coupling strength depends on the position and the mode we talk to



Coupling is maximal close to the borders because charge accumulates on capacitors

2) Coupling depends on frequency: $g \propto \sqrt{\omega}$

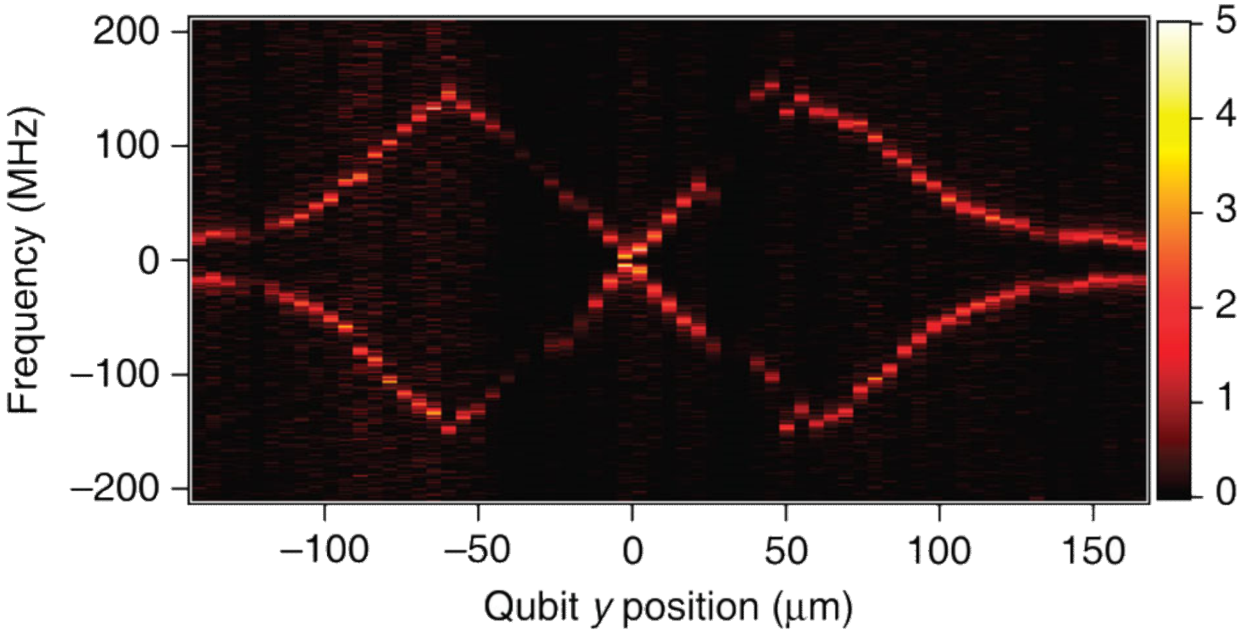
$\hookrightarrow J(\omega) = \pi \alpha \omega$ once more!

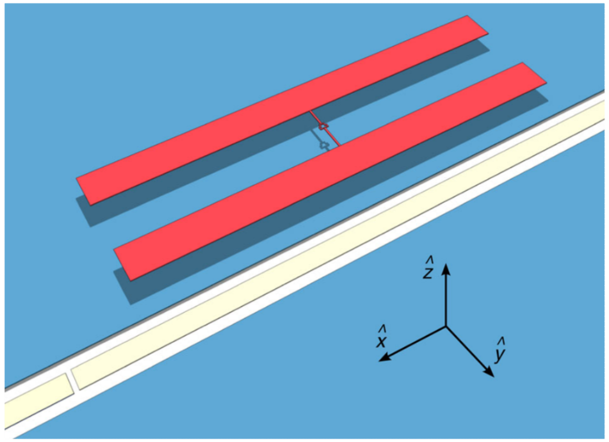
↳ $J(\omega) = \pi \alpha \omega$ once more!

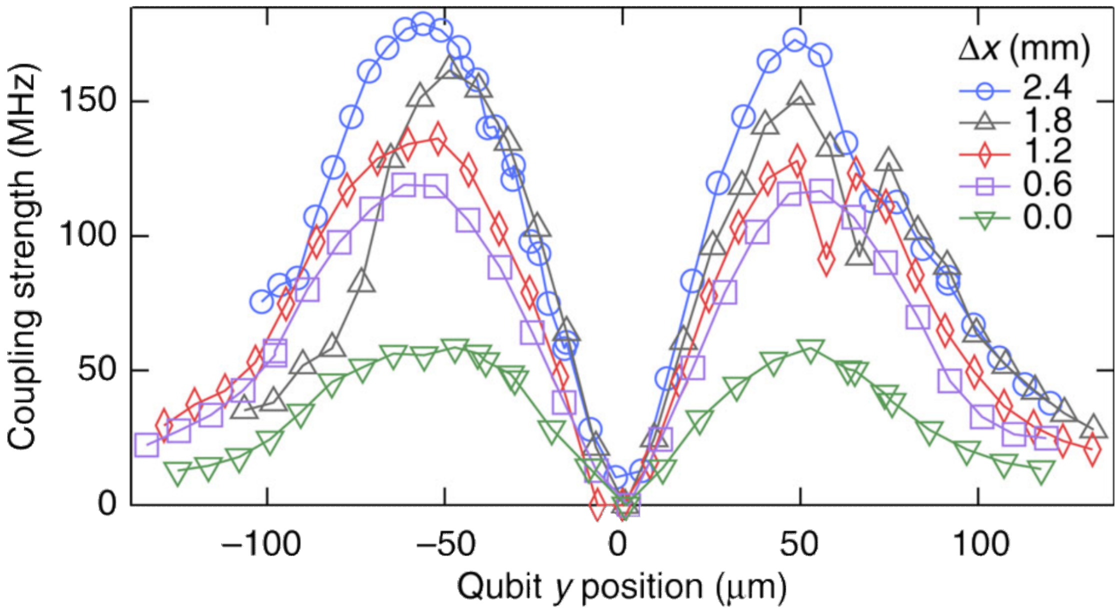
↳ Important also on multimode lines, where we can shift the qubit to talk to upper or lower modes

Scanning qubit

Tuesday, February 16, 2016 9:30 PM







A scanning transmon qubit for strong coupling circuit quantum electrodynamics
W. E. Shanks, D. L. Underwood &A. A. Houck, Nat. Comm. 4, 1991 (2013)

[This is a transmon, but physics is the same, as we will see later]