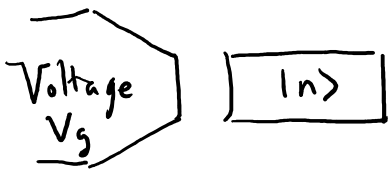
Charge as quantum degree of freedom

Wednesday, February 10, 2016 2:18 PM

* Let us take a superconducting island. It's charging energy provides with an anharmonicity and a qubit design

$n = \#$ excess cooper pairs $(-q/2e)$

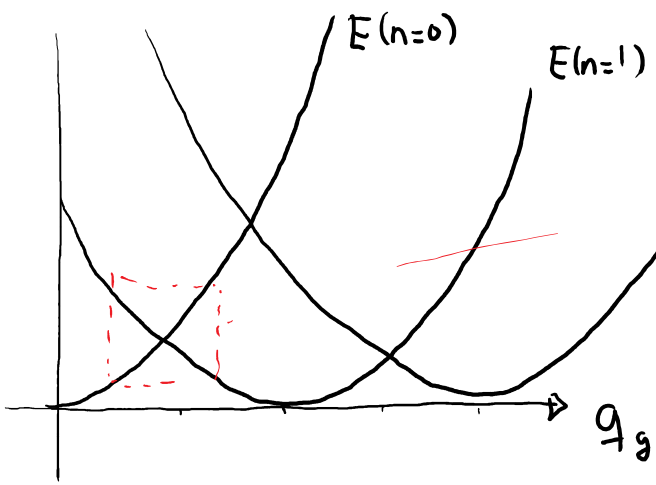


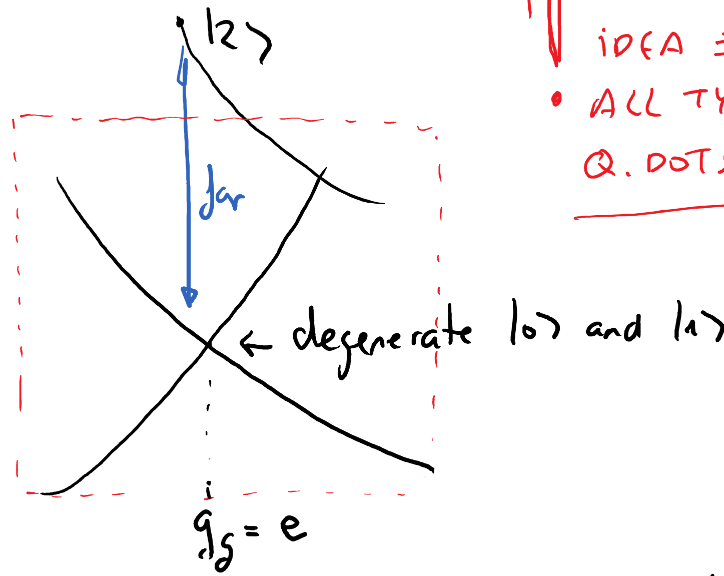
$E \sim \frac{1}{2C} q^2 - q\,U \qquad U \propto V_g$

$E \sim \frac{1}{2C} (q - q_g)^2$

NOT A SUPERC. IDEA ⇒ SEE

• ALL TYPES OF Q. DOTS !!!



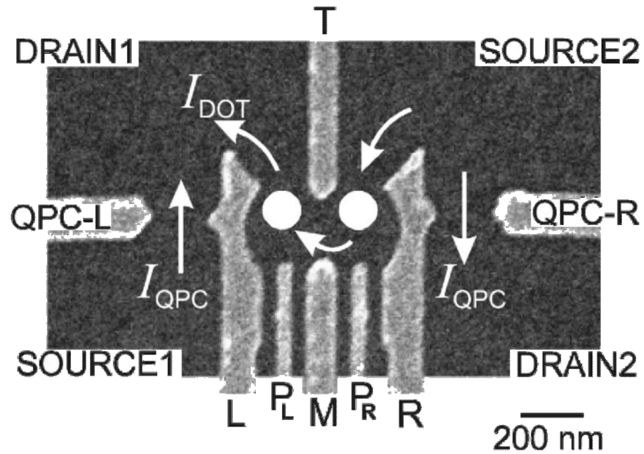


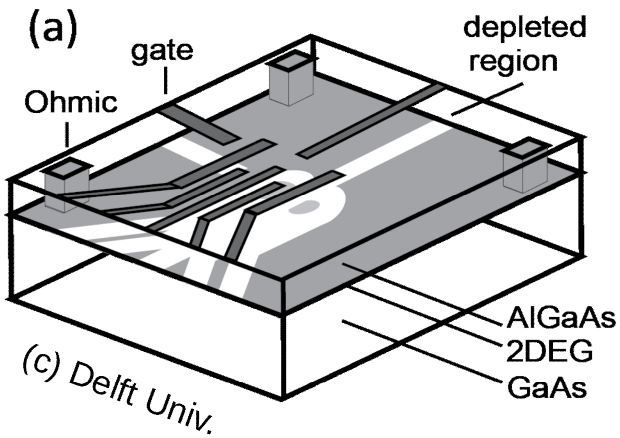
* The problem is that we do not only need qubit states: we also need transitions

Not a superconductor thing

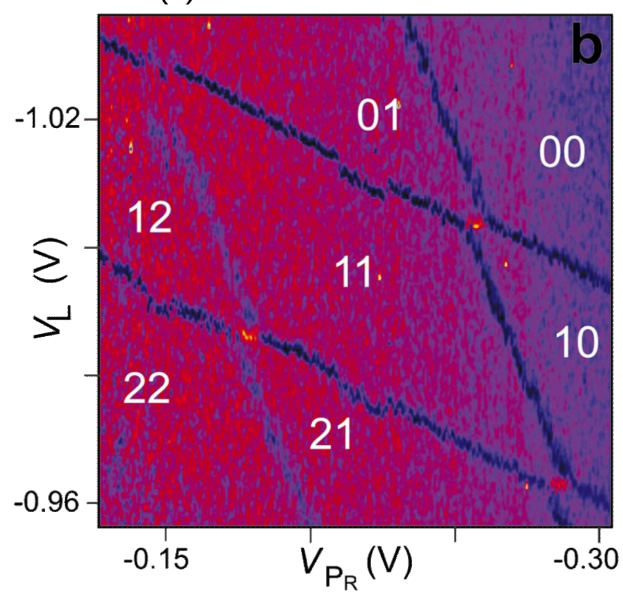
Monday, February 15, 2016 9:10 AM

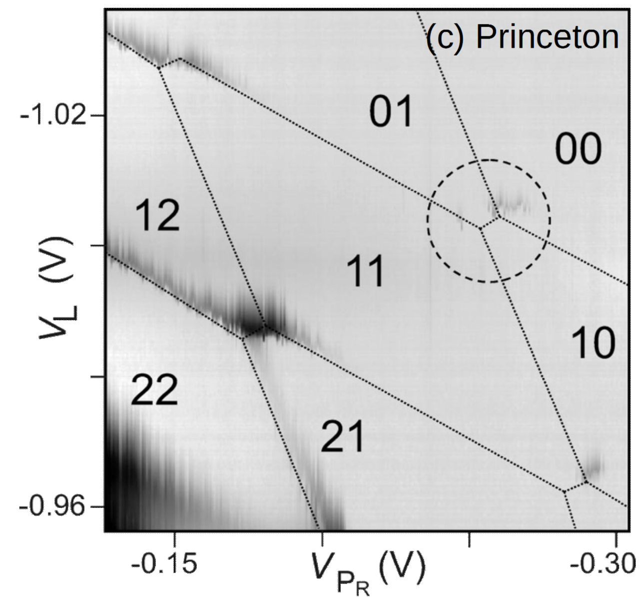
2D
electron gas
quantum
dots
:-)





(c) Princeton

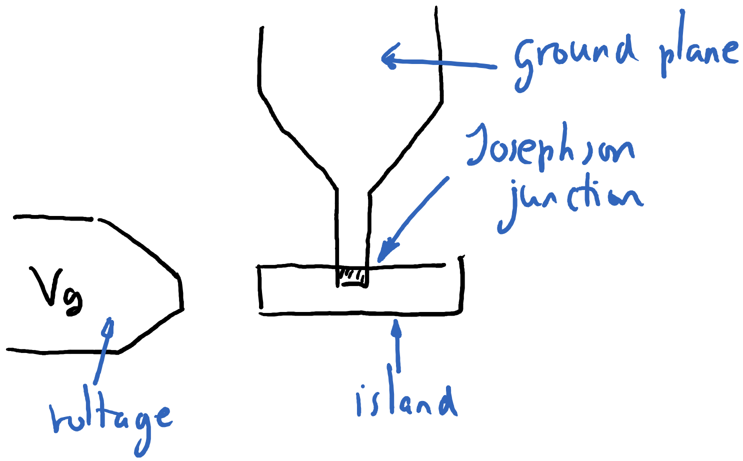




Only charge quantization and Coulomb blockade are required

Improved design: charge qubit

Wednesday, February 10, 2016 2:28 PM



* We allow charges to tunnel in and out of the island through the junction
  ↳ quantum fluctuations!
* We treat the ground plane as a coherent state which is not affected by tunneling of particles
  ↳ no decoherence induced by reservoir

* Already with this we can write an effective Hamiltonian that captures the Physics:

$$H \sim \frac{1}{2C} q^2 - q\,U + \frac{\varepsilon}{2} \sum_n \left(|n+1\rangle\langle n| + H.c. \right)$$

* Around the degenerate points ($q_g = 1/2$)

$$H \sim \left[q_g - 1/2 \right] \left[-|1\rangle\langle 1| + |0\rangle\langle 0| \right] + \frac{\varepsilon}{2} \left(|1\rangle\langle 0| + |0\rangle\langle 1| \right)$$

$$\sim \frac{\Delta}{2} \sigma^z + \frac{\varepsilon}{2} \sigma^x$$

Circuit theory

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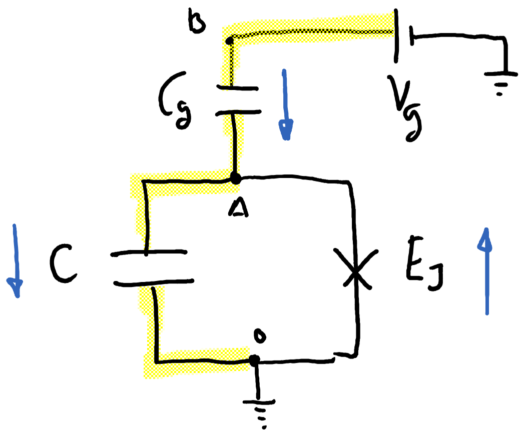


$$I_J = I_c - I_V$$

$$I_c = C\,(\ddot{\phi}_A - \ddot{\phi}_0)$$

$$I_J = I_c \sin\left([\phi_0 - \phi_A]\frac{2\pi}{\Phi_0}\right)$$

$$I_V = C_g\,(\ddot{\phi}_B - \ddot{\phi}_A)$$

* We use:
  a) No flux trapping
  b) $\dot{\phi}_B = V_g$
  c) $\phi_0 = 0$ (ground plane)

* Differential equation $\quad -I_c \sin\left(\phi_A \frac{2\pi}{\Phi_0}\right) = C\,\ddot{\phi}_A + C_g(\ddot{\phi}_A - \dot{V}_g) = \frac{d}{dt}\left[C_\Sigma\left[\dot{\phi}_A - \frac{C_g}{C_\Sigma}V_g\right]\right]$

$$C_\Sigma = C + C_g$$

Lagrangian $\quad \mathcal{L} = \frac{1}{2}C_\Sigma\left[\dot{\phi}_A - \frac{C_g}{C_\Sigma}V_g\right]^2 + I_c\frac{\Phi_0}{2\pi}\cos\left(\phi_A\frac{2\pi}{\Phi_0}\right)$

Charge: $\quad q = C_\Sigma\dot{\phi}_A - C_g V_g = C_\Sigma\dot{\phi}_A + q_g$

Hamiltonian $\quad \mathcal{H} = \frac{1}{2C_\Sigma}(q - q_g)^2 - I_c\cdot\frac{\Phi_0}{2\pi}\cos\left(\frac{2\pi}{\Phi_0}\phi_A\right)$

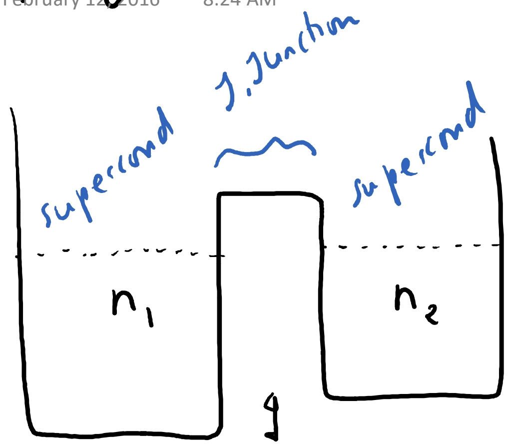
$\to E_J$ (Josephson energy)

What are the eigenstates?

How do we solve this?

Hopping model

Friday, February 12, 2016 8:24 AM



they are bosonic particles!!!

$\bar{N}_0$ charge neutrality

$$H \sim \frac{V}{2} n_2 - \frac{V}{2} n_1 - t\left(a_2^\dagger a_1 + a_1^\dagger a_2\right) + 4E_c (n_2 - \bar{N}_0)^2 + 4E_c (n_1 - \bar{N}_0)^2$$

Population imbalance states

$$|n_1, n_2\rangle \to |\bar{N}, \delta n\rangle \quad \begin{cases} \bar{N} = \dfrac{n_1 + n_2}{2} \\ \delta n = (n_1 - n_2)/2 \end{cases}$$

Large number approximation

$$a_1^\dagger a_2 \sim \sum_{n_1, n_2} \sqrt{n_2 (n_1 + 1)}\, |n_1 + 1, n_2 - 1\rangle\langle n_1, n_2|$$

$$\sim \bar{N} \sum_{\delta n} |\delta n + 1\rangle\langle \delta n|$$

$$H \sim \text{constants} - V \cdot \hat{\delta n} + 2E_c (\hat{\delta n})^2 - t \sum_{\delta n} \left(|\delta n + 1\rangle\langle \delta n| + Hc\right)$$

...

$$(n_2 - \bar{N}_0)^2 + (n_1 - \bar{N}_0)^2 \sim \left(\bar{N} - \bar{N}_0 - \frac{\delta n}{2}\right)^2 + \left(\bar{N} - \bar{N}_0 + \frac{\delta n}{2}\right)^2 \sim 2(N - \bar{N}_0)^2 + \frac{(\delta n)^2}{2}$$

Phase number representation

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$$|\varphi\rangle = \frac{1}{\sqrt{2\pi}} \sum_{n\in\mathbb{Z}} e^{-i\varphi n} |n\rangle \quad \Rightarrow \quad |n\rangle = \int_0^{2\pi} d\varphi \, \frac{e^{+i\varphi n}}{\sqrt{2\pi}} |\varphi\rangle \, , \quad \langle n|m\rangle = \int_0^{2\pi} \frac{e^{-i\varphi(n-m)}}{2\pi} d\varphi = \delta_{nm}$$

$$\langle\varphi|\zeta\rangle = \sum_n \delta(\varphi - \zeta + 2\pi n)$$

Write down a wavepacket $|\Psi\rangle = \int_0^{2\pi} \Psi(\varphi)|\varphi\rangle d$, $\psi(2\pi) = \psi(0)$, continuous...

↳ Number operator $\hat{N}|\Psi\rangle = \int_0^{2\pi} \Psi(\varphi) \frac{1}{\sqrt{2\pi}} \sum_n e^{-i\varphi n} n |n\rangle = \int_0^{2\pi} \left(-i\partial_\varphi \Psi(\varphi)\right) |\varphi\rangle d\varphi$

EXACT

↳ $S^+ = \sum_n |n+1\rangle\langle n| \Rightarrow S^+|\Psi\rangle = \int_0^{2\pi} \Psi(\varphi) \frac{1}{\sqrt{2\pi}} \sum_n e^{-i\varphi n} |n+1\rangle = \int_0^{2\pi} e^{i\varphi n} \Psi(\varphi)|\varphi\rangle d\varphi$

FLUCTUATIONS << mean N

OK AS WE SAW FROM LC RESONATORS!

$$\hat{N} \leftrightarrow -i\partial_\varphi , \quad S^+ \leftarrow e^{i\varphi n}$$

↳ Charge qubit : $H = 4E_c(\hat{N} - n_g)^2 - \underbrace{t(S^+ + S^-)}_{t(e^{i\varphi} + e^{-i\varphi})} \sim 4E_c(-i\partial_\varphi - n_g)^2 - \overset{2t}{\overset{\parallel}{E_J}} \cos(\varphi)$

Qubit Hamiltonian

Saturday, February 13, 2016 10:28 AM

$$H \simeq 4E_c(\hat{n}-n_g)^2 - \frac{E_J}{2}\left(\sum_n |n+1\rangle\langle n| + H.c\right)$$

Around degeneracy $n_g \sim 1/2 + \delta n$

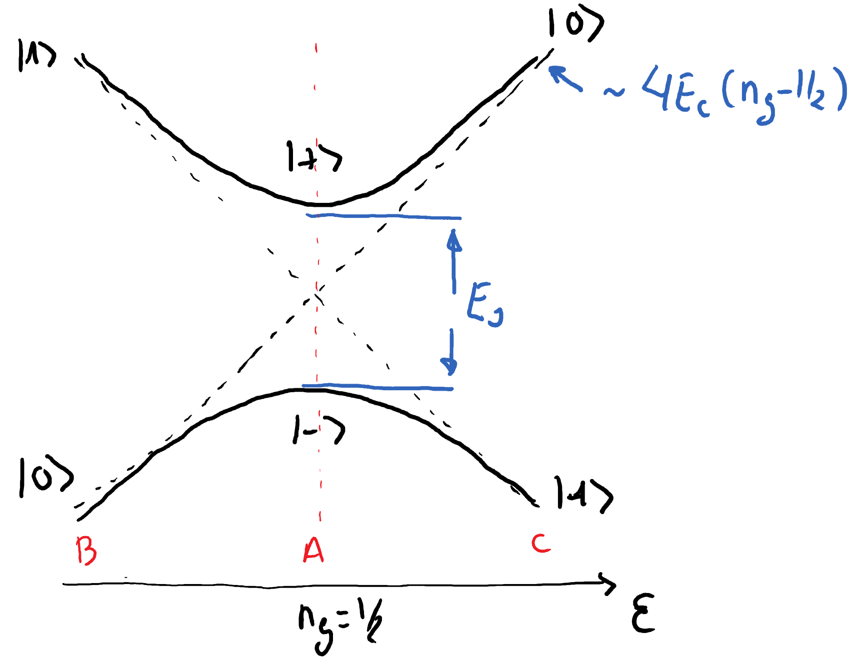
$$H \sim 4E_c\left(\tfrac{1}{2}\right)^2 - \delta n\, 8E_c\left(\hat{n}-1/2\right) + \frac{E_J}{2}\left(s^+ + s^-\right)$$

Pauli operators : $\sigma^z = |1\rangle\langle 1| - |0\rangle\langle 0| = 2(\hat{n}-1/2)$

$\sigma^x = |1\rangle\langle 0| + |0\rangle\langle 1| = s^+ + s^-$

$$H \sim \frac{E_J}{2}\sigma^x - \frac{\varepsilon}{2}\sigma^z \qquad \varepsilon \sim 16\,E_c\,(n_g - 1/2)$$

$\sigma^z \propto$ dipole moment !



Eigenstates

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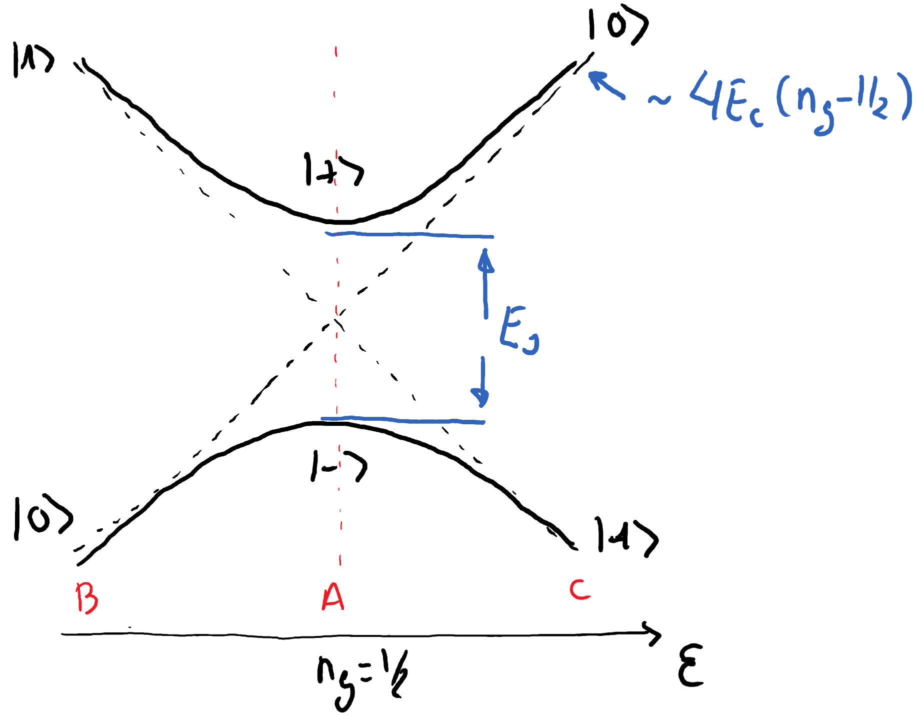


A : $H \sim -\frac{E_c}{2}\sigma^x$

Ground state $|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

Excited state $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

B : $H \sim \frac{}{2}\sigma^z = \frac{}{2}(|1\rangle\langle 1| - |0\rangle\langle 0|)$

excited state

ground state

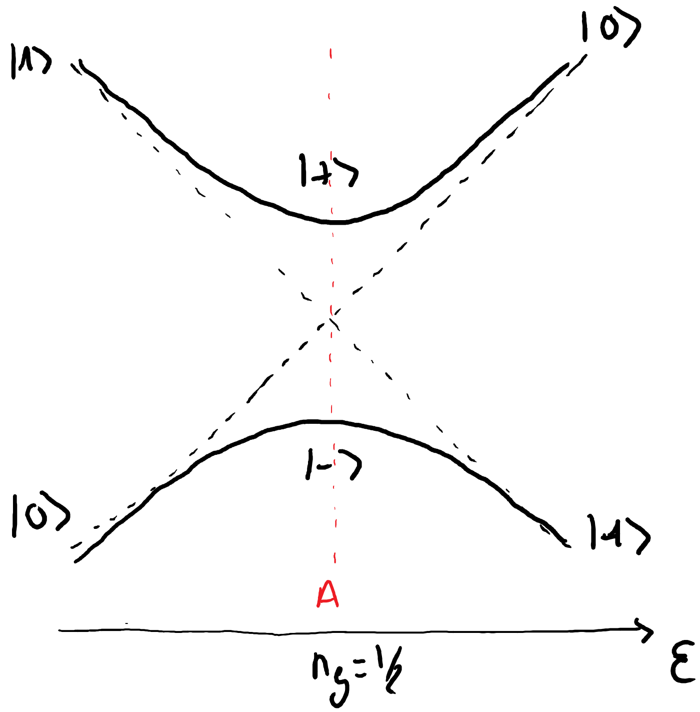
C : $|0\rangle$ ~ excited state, $|1\rangle$ - ground state

In general $H = -\frac{E_c}{2}\sigma^x + \frac{\varepsilon}{2}\sigma^z$

$\hookrightarrow E_\pm = \pm\frac{1}{2}\sqrt{E_c^2 + \varepsilon^2}$ "qubit hyperbola"

Qubit eigenbasis

Saturday, February 13, 2016 5:08 PM



* The symmetry point (A) is the point at which external fields have the least influence on the eigenenergie
  ↳ $\left.\frac{dE_\pm}{d\varepsilon}\right|_{\varepsilon=0} = 0$ } only 2nd order dephasing
* It is also the point where spontaneous emission is slowest ($\Gamma_1 \propto$ gap)
* The symetry point is the desired working point for many qubits : why not define qubit states there?

* Usual conventions : $|+\rangle, |-\rangle \rightarrow |\tilde{1}\rangle, |\tilde{0}\rangle \rightarrow H \sim \frac{\Delta}{2}\tilde{\sigma}^z + \frac{\varepsilon}{2}\sigma^x$
  ↳ $\sigma^x \propto$ electric dipole or charge ↳ $\varepsilon(t)$ ~ external driving source
  ↳ $\Delta \sim E_J \sim$ qubit "gap"

Qubit control

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a) Cooling : set up a field and wait for the qubit to relax

$$\omega \sim 2-9 \text{ GHz} \gg k_B T$$

$$P_e \sim \exp\left(-\hbar\omega / k_B T\right) \ll 1$$

b) Adiabatic passage : move from one ground state to another by slowly changin $\begin{cases} E_J \\ \varepsilon \end{cases}$

↳ If $\hbar\Delta(t)$ is the instantaneous gap,

$$v = \frac{1}{\Delta} \cdot \frac{d\Delta}{dt} \quad \text{the speed}$$

$$\tau \sim \frac{1}{v} \quad \text{the time scale for change}$$

$$\left.\right\} \quad \tau \gg \frac{1}{\Delta} \Rightarrow \text{adiabatic condition}$$

↳ $H = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon}{2}\sigma^x, \Rightarrow \frac{\dot{\varepsilon}(t)}{\varepsilon} \ll \Delta$

c) Diabatic or instantaneous change : just the opposite! $\quad \tau \ll \frac{1}{\min(\text{gap})}$

Qubit control (2)

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d) Free evolution : $H = \frac{\Delta}{2}\sigma^z + \frac{\varepsilon}{2}\sigma^x$ generates some rotation

$$i\frac{d}{dt}|\psi\rangle = H|\psi\rangle \Rightarrow |\psi(t)\rangle = e^{-iH(\Delta,\varepsilon)t/\hbar}|\psi(0)\rangle$$

↳ This does not include all single-qubit rotations $U = \exp(i\vec{n}\cdot\vec{\sigma}\cdot\theta)$

↳ They can only be obtained by composing gates

↳ Only σ^z is obtained exactly ($\varepsilon = 0$)

↳ σ^x rotations only approximate ($|\varepsilon| \gg |\Delta|$) ⇒ too large fields enhance decoherence!

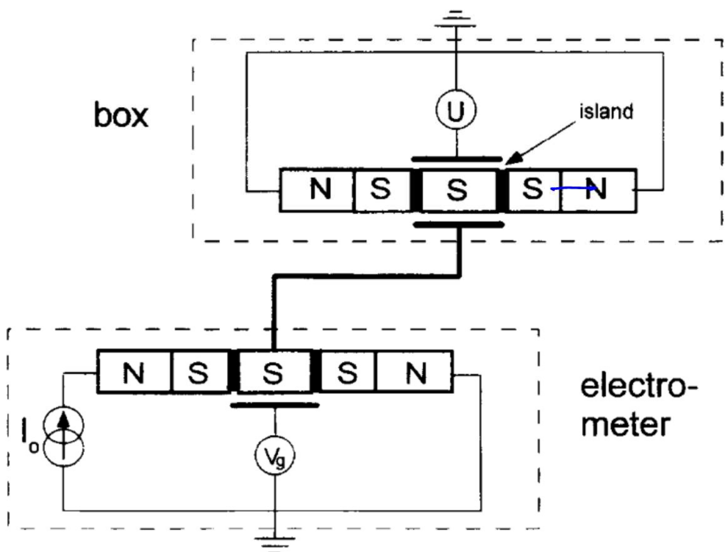
e) External driving : make Δ or ε depend on time to engineer arbitrary $U(t)$

End of lecture Feb 15th

First charge qubit experiments

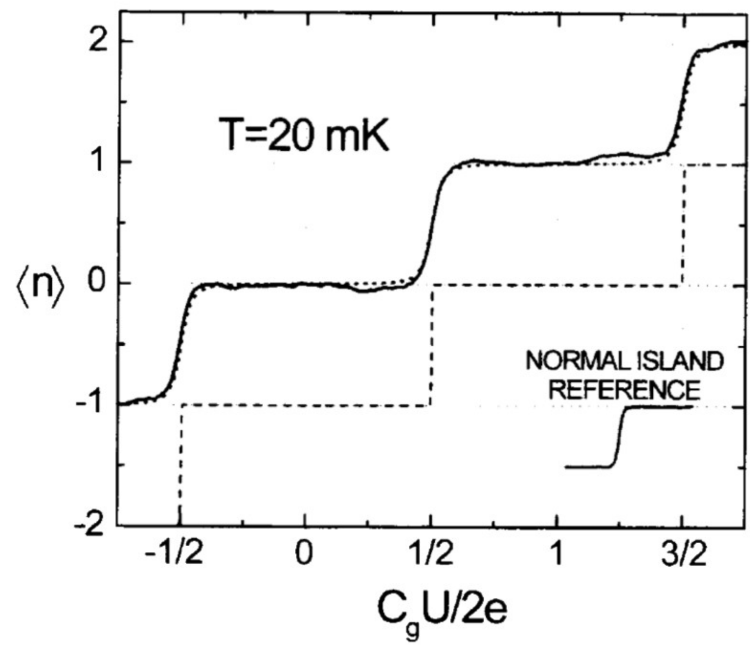
Saturday, February 13, 2016 6:01 PM

* The device consists on a charge qubit electrostatically coupled to a single electron transistor, capable of measuring "Q" in the qubit



* Note that this expt. is completely "classical" and does not prove quantum coherence, only equilibrium configurations compatible with charge quantization

Bouchiat, Vincent, D. Vion, Ph Joyez, D. Esteve, and M. H. Devoret. "Quantum coherence with a single Cooper pair." *Physica Scripta* 1998, no. T76 (1998): 165



Proof of coherence

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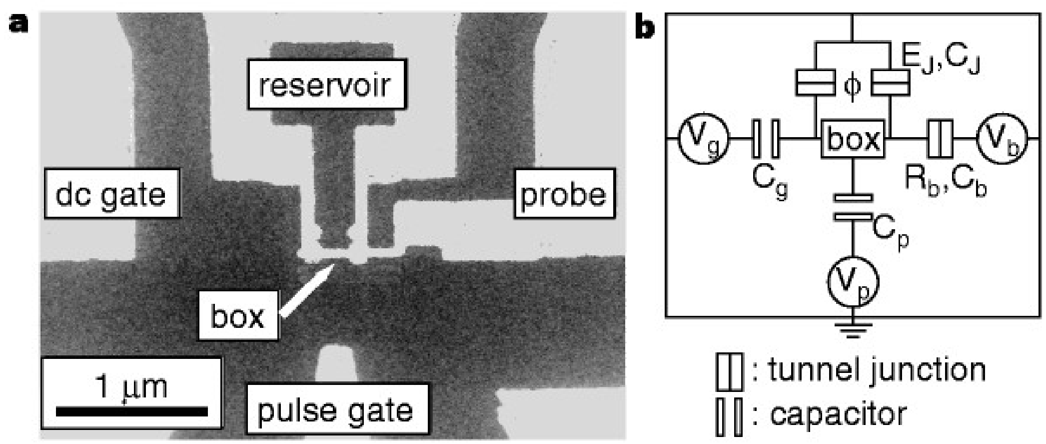
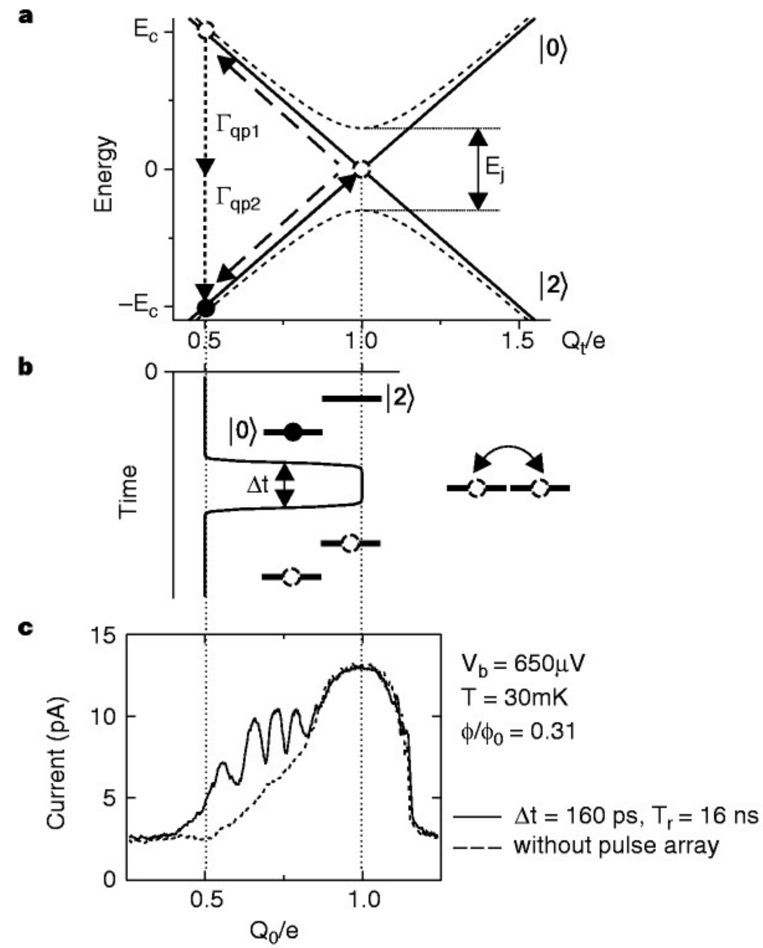


[Coherent control of macroscopic quantum states in a single-Cooper-pair box]
Y. Nakamura, Yu. A. Pashkin and J. S. Tsai *Nature* **398**, 786-788(29 April 1999)



Idea: start with a well defined charge state, jump to degeneracy and see oscillations in charge

From
<http://www.nature.com/nature/journal/v398/n6730/fig_tab/398786a0_F2.html>

Idea: coherent oscillations

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Protocol:

a) Apply a large external field and cool to zero charge $H_{T=0} \approx +\frac{\varepsilon_\infty}{2}\sigma^z$

b) Switch off ε for a brief period of time and evolve with $H_0 = \frac{\Delta}{2}\sigma^x$

c) Go back to large ε and measure "g" or "n"

$$|\psi(0)\rangle = |0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle e^{-i\Delta t/2} + |-\rangle e^{+it\Delta/2}\right)$$

$$= \frac{1}{\sqrt{2}}\left[|0\rangle \cos\frac{\Delta t}{2} - i\sin\left(\frac{\Delta t}{2}\right)|1\rangle\right]$$

$$\langle g(t)\rangle \sim \cos\left(\frac{\Delta t}{2}\right)$$

Dependence on waiting time

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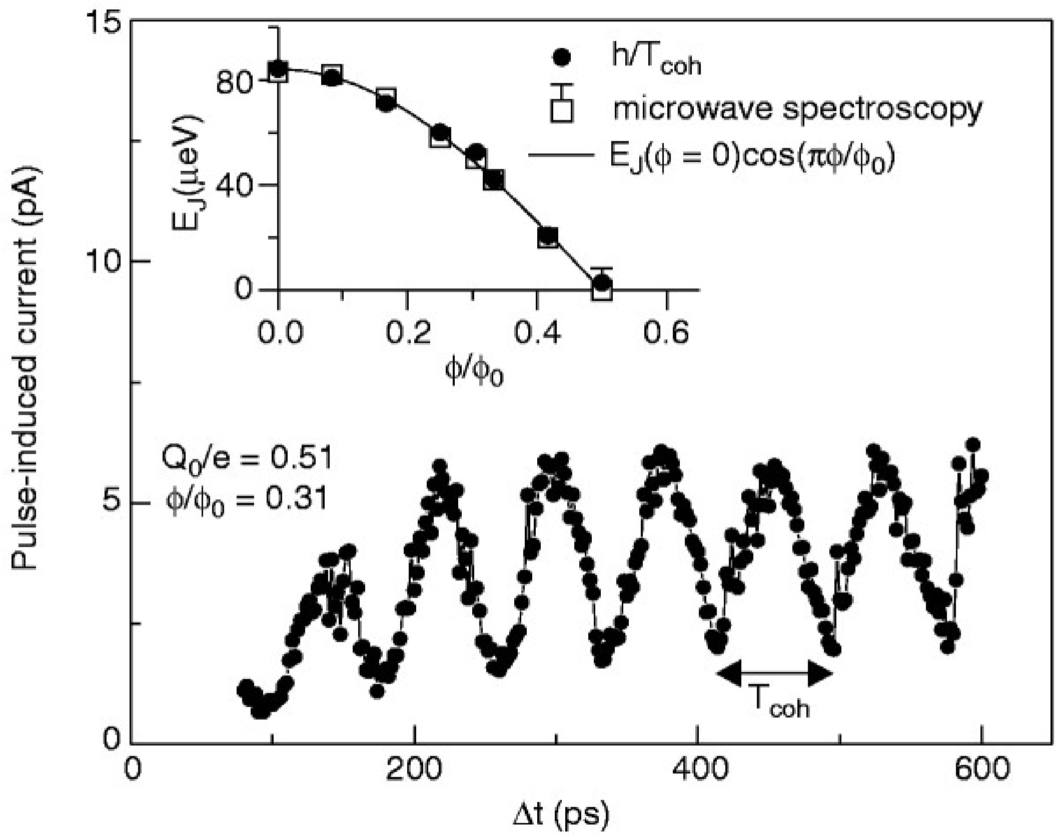


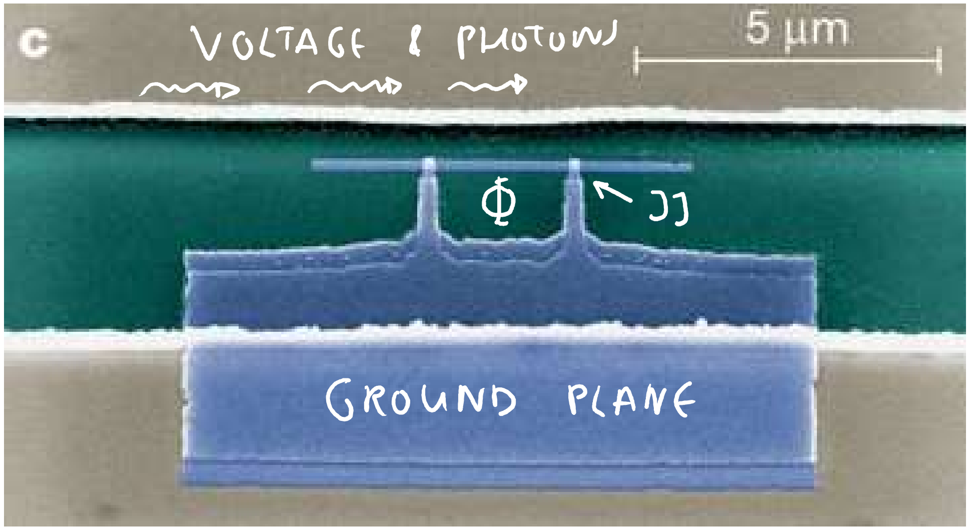
Coherent control of macroscopic quantum states in a single-Cooper-pair box
Y. Nakamura, Yu. A. Pashkin and J. S. Tsai *Nature* **398**, 786-788(29 April 1999)

$\frac{1}{\Gamma} \sim 2$ ns

very bad coherence time !!!

A state-of-the-art charge qubit

Wednesday, February 10, 2016 3:30 PM



Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics
A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.- S. Huang, J. Majer, S. Kumar, S. M. Girvin and R. J. Schoelkopf
Nature **431**, 162-167(9 September 2004)

* Charge qubit embedded in microwave resonator
  ↳ longer lifetime

  $\gamma / 2\pi \sim 0.7$ MHz

* Comparable energy scale

  $E_c \approx h \times 5.2$ GHz $\quad (4E_c >> E_J)$

  $E_J \approx h \times 8.1$ GHz

* tunneling is actually controlled by the loop flux Φ (→ lectures on SQUIDs)

  $E_J \sim \cos(\Phi) \cdot E_J^{max}$