Day 3

Recap

Quantum fluid model: charged bosons coupled to EM field

$$i\hbar\partial_t \psi = \left\{ \frac{1}{2m}\left[-i\hbar\vec{\nabla} - q\vec{A}\right]^2 + qU \right\}\psi$$

↳ Wavefunction: $\psi \sim \sqrt{n}\, e^{i\theta}$, n: charge carrier dty

↳ 1st London equation $\quad \frac{\partial}{\partial t}(\Lambda\vec{J}) = \vec{E} - \frac{1}{nq}\vec{\nabla}\left(\frac{\Lambda}{2}\vec{J}^2\right)$

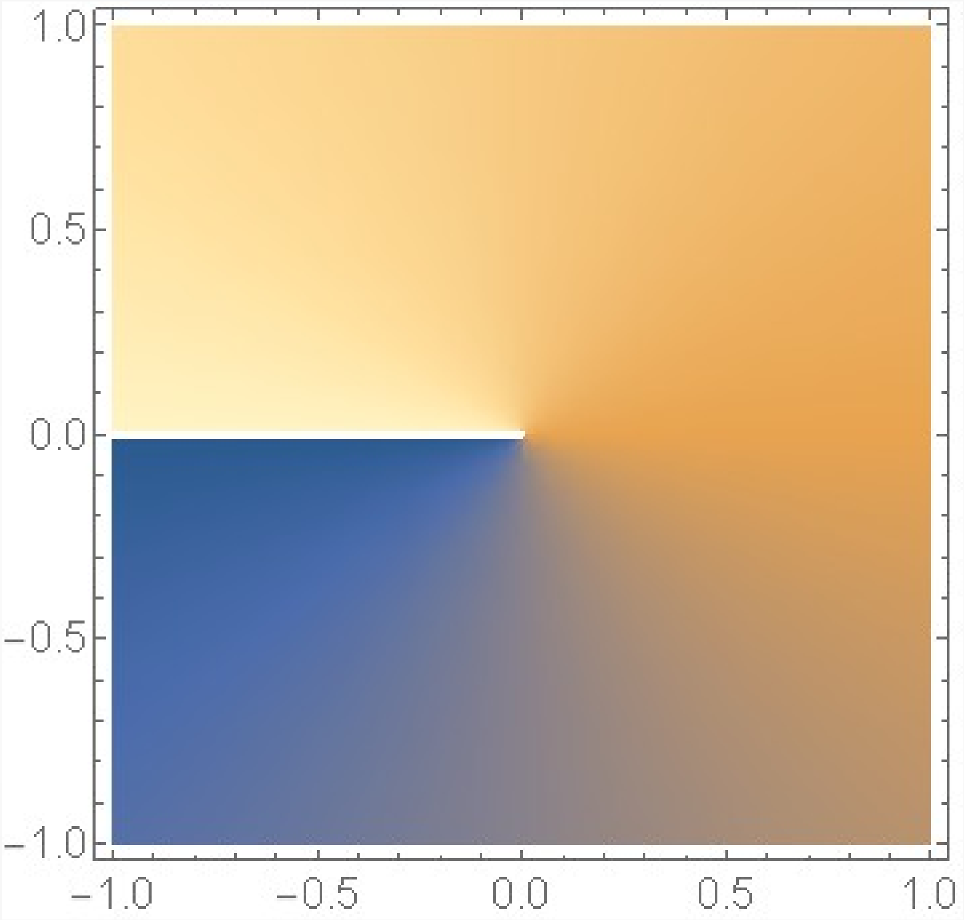
↳ $\Lambda = \frac{m}{q^2 n}$ ~ inductance $\quad \sqrt{\frac{\mu}{\Lambda}}$ ~ penetration depth

↳ Charge current $\quad \vec{J} \sim qn\left[\frac{\hbar}{m}\vec{\nabla}\theta - \frac{q}{m}\vec{A}\right]$

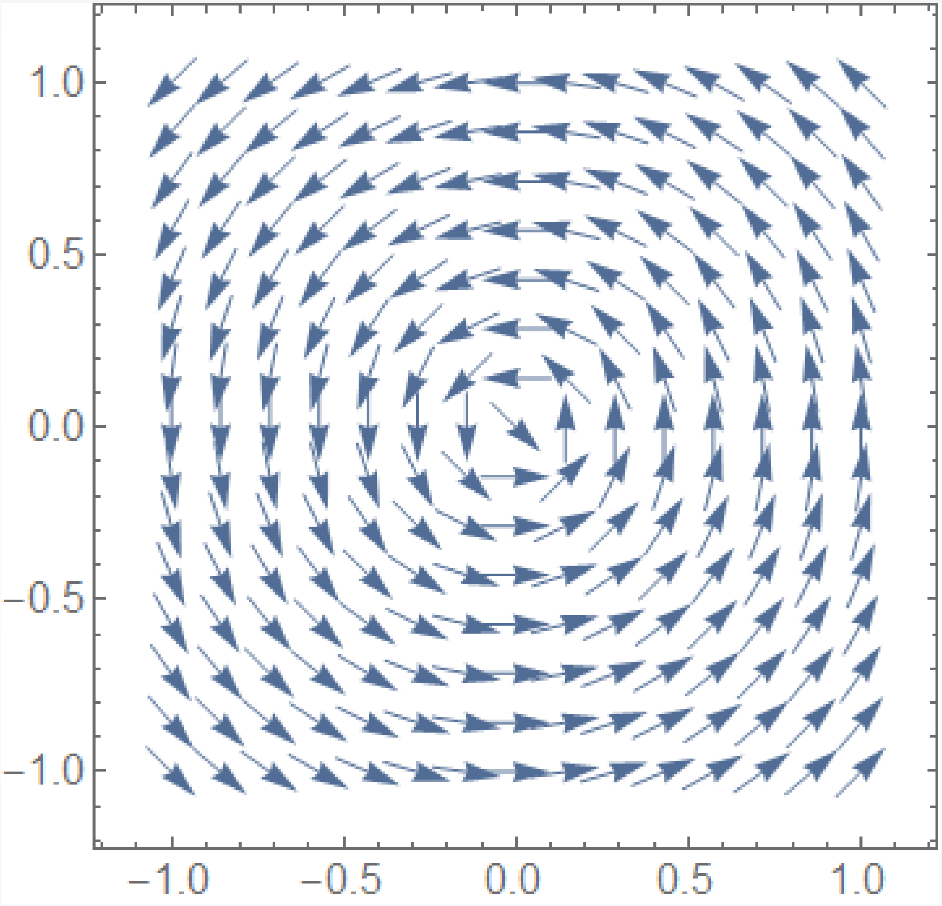
↳ Fluxoid quant. $\quad \oint_C (\Lambda\vec{J})\cdot d\vec{l} + \underbrace{\oint_C \vec{B}\cdot d\vec{s}}_{\Phi_{ext}} \in \Phi_0 \times \mathbb{Z} \qquad \Phi_0 = \frac{h}{q} = \frac{h}{2e}$ flux quantum

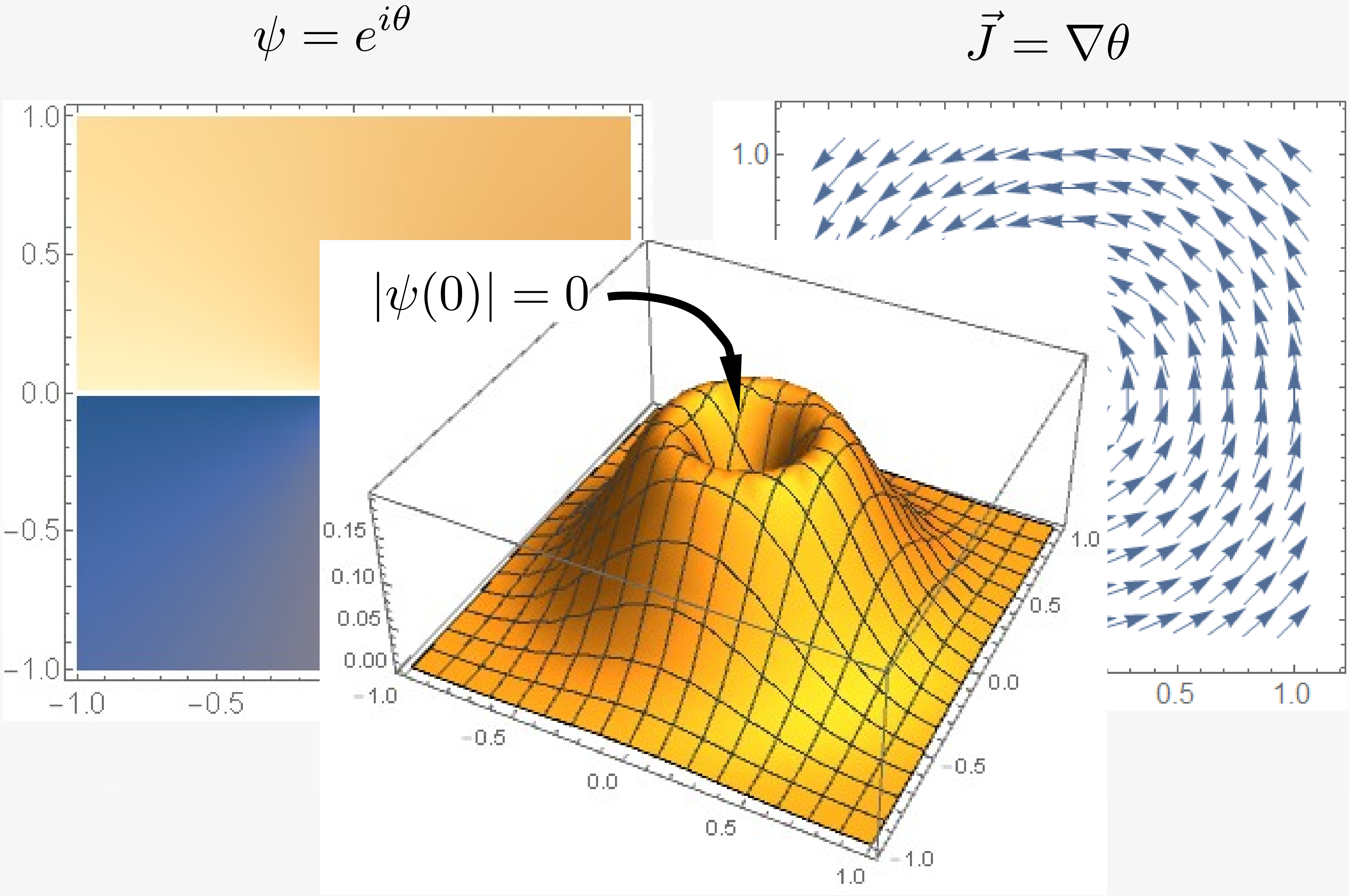
↳ Flux-phase $\quad \partial_t\theta = -\frac{qU}{\hbar}$

$$\psi = e^{i\theta}$$

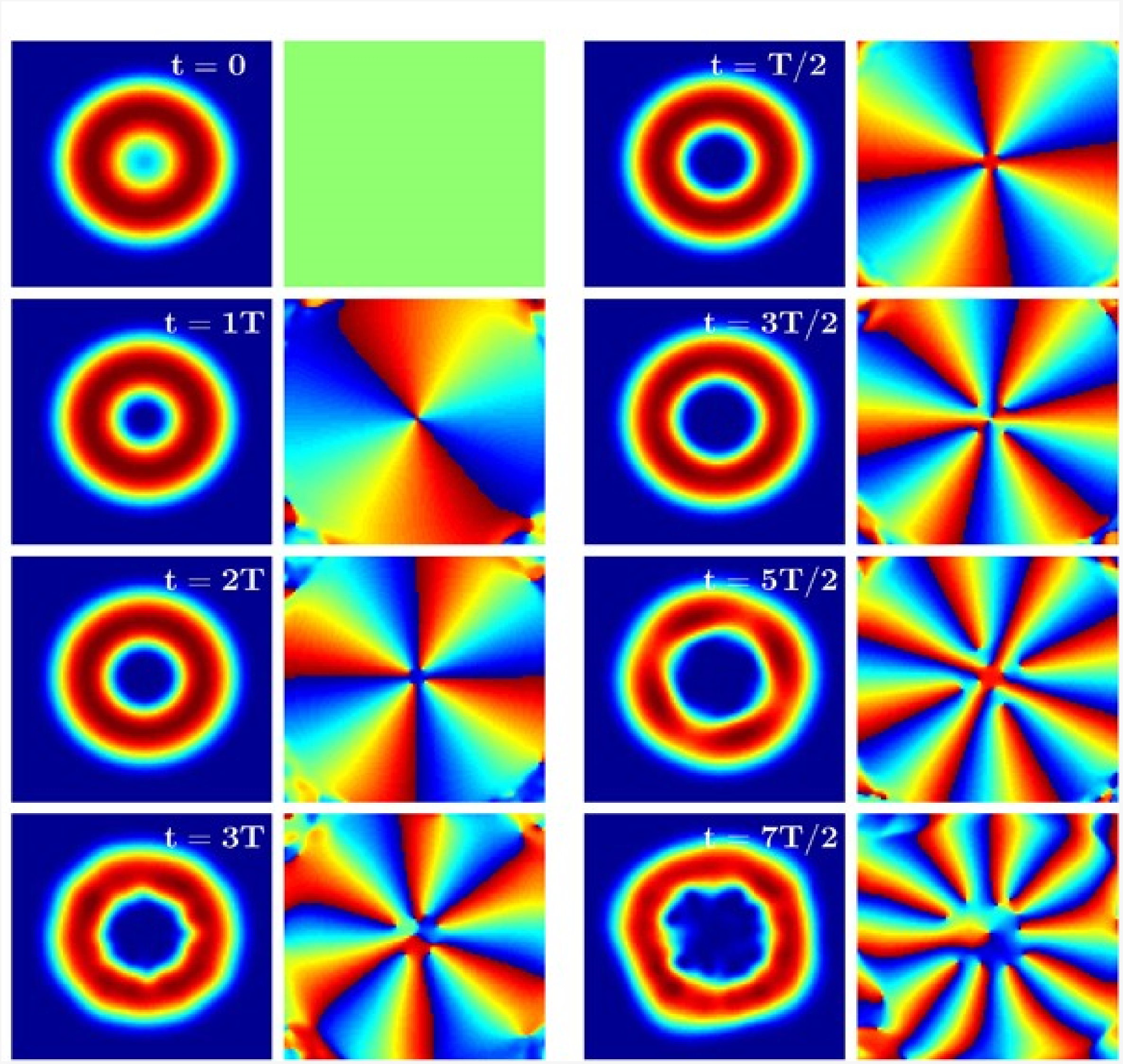


$$\vec{J} = \nabla\theta$$




$\psi = e^{i\theta}$
$\vec{J} = \nabla\theta$
$|\psi(0)| = 0$

Higher quantum numbers



(c) Univ Aalto

Directed vs. trapped current